Dynamic Equilibrium at a Congestible Facility under Market Power

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Background

* Economics of airport congestion vs road congestion
  * Market power: self-internalization
    * Daniel (1995); Brueckner (2002); Pels and Verhoef (2005)
  * Pigouvian τ = *mec*: overcharging
* Mixed empirical evidence
    * Daniel and Harback (2008); Mayer and Sinai (2003)
* Consistent with further theoretical analyses
  * Stackelberg (Brueckner and Van Dender, 2008)
  * Bertrand (Silva and Verhoef, 2013)

Recently: dynamics of congestion

- Vickrey bottleneck with oligopolistic operators
  - Silva, Verhoef and Van den Berg (2014)
  - Leader - fringe: leader is forced to schedule according to atomistic patterns in the peak center
  - Cournot – Nash: no equilibrium
    - Only candidate equilibrium in which unilateral *marginal* changes in arrival flows do not increase profits fails, because a *non-marginal* change increases profits
  - Silva, Lindsey, De Palma and Van den Berg (2014)
    - Confirm non-existence for $\alpha < \gamma$
    - Nash equilibrium for $\alpha > \gamma$, but: queue free!

This paper

* Discomforting: no dynamic equilibrium with visible congestion
  * Due to dynamics, or to congestion technology?
  * Some alternatives to Vickrey (flow congestion)
    * Continuous-time – continuous-space (kinematic, car-following)
    * Henderson (1974, 1981) – Chu (1995): “no propagation”
    * Agnew (1977): “infinite propagation”
* Does this
  * Provide a dynamic equilibrium?
  * If so, how efficient is it?

Chu (1995) in brief

- Demand side
  - Fixed number of identical travellers *N*
  - "αβγ-preferences"
- Supply side
  - Travel delay is a function alone of the instantaneous arrival rate *f* at the moment of arriving
  - Chu uses BPR-type of function

$$T(f(t);K) = T_f + \left(\frac{f(t)}{K}\right)^{\chi}$$

Atomistic equilibrium and optimum

- Closed form
- Optimal time-varying toll takes on Pigouvian form:

$$\tau(t) = f(t) \cdot \frac{\partial c_T(f(t))}{\partial f(t)}$$

- Optimum versus atomistic equilibrium
  - Wider arrival interval
  - Lower flows
    - Illustration will follow

Chu with Cournot operators

- Good to know: whether congestion is incurred by passengers (disutility of delays) or operators (crew cost), it enters profit problem symmetrically
  - Higher passenger congestion: lower WTP for fare
- We thus work with a single congestion cost function
  - Ignore all other time-independent cost
- Focus on case with two firms
- Relevant cost functions
  - Average cost ac: travel delay + schedule delay
  - Firm-internal marginal cost mc_i: ac + firm-internal congestion externality

In maths:

- General model

$$ac_i(t) = ac_j(t) = c_T(f_i(t) + f_j(t); K) + c_{SD}(t)$$

$$mc_x(t) = c_T(f_i(t) + f_j(t); K) + f_x(t) \cdot c_T{}'(\cdot) + c_{SD}(t) \quad x = \{i, j\}$$

- Specific model (αβγ-preferences; BPR delay function)

$$ac_i(t) = ac_j(t) = \alpha \cdot \left(\frac{f_i(t) + f_j(t)}{K} \right)^{\chi} + \begin{cases} -\beta \cdot t & \text{if } t \le 0 \\ \gamma \cdot t & \text{if } t > 0 \end{cases}$$

$$mc_x(t) = \alpha \cdot \left(\frac{f_i(t) + f_j(t)}{K} \right)^{\chi} + f_x(t) \cdot \chi \cdot \frac{1}{K} \cdot \left(\frac{f_i(t) + f_j(t)}{K} \right)^{\chi - 1} + \begin{cases} -\beta \cdot t & \text{if } t \le 0 \\ \gamma \cdot t & \text{if } t > 0 \end{cases} \quad x = \{i, j\}$$

Nash equilibrium

* Minimize firm-internal cost by choosing arrival schedule, treating competitor's schedule as fixed
* Consequence: mc_i is constant over time in the firm's arrivals window(s), and at least as high outside
  * Time derivatives of mc_i are zero in intervals
* Two useful features of equilibrium:
  1. It cannot entail disjoint monopolized intervals
  2. If intervals do not overlap exactly, we have a sequence:

$$i \quad - \quad i \,\&\, j \quad - \quad i$$

$$t^* = 0$$

Arrival patterns

- With asymmetric firms, *i* internalizes all congestion when travelling alone in the shoulders of the peak
- Trouble starts when they travel jointly in the center

$$m\dot{c}_i(t) = \dot{c}_{SD}(t) + \dot{f}(t)\cdot c_T{}'(\cdot) + \dot{f}_i(t)\cdot c_T{}'(\cdot) + f_i(t)\cdot \dot{f}(t)\cdot c_T{}''(\cdot) = 0 \quad \forall t: t_{qj} \leq t \leq t_{ej}$$

$$m\dot{c}_j(t) = \dot{c}_{SD}(t) + \dot{f}(t)\cdot c_T{}'(\cdot) + \dot{f}_j(t)\cdot c_T{}'(\cdot) + f_j(t)\cdot \dot{f}(t)\cdot c_T{}''(\cdot) = 0 \quad \forall t: t_{qj} \leq t \leq t_{ej}$$

- For general case: *j* catches up but never exceeds *i*:

$$\dot{f}_i(t)\cdot c_T{}'(\cdot) + f_i(t)\cdot \dot{f}(t)\cdot c_T{}''(\cdot) = \dot{f}_j(t)\cdot c_T{}'(\cdot) + f_j(t)\cdot \dot{f}(t)\cdot c_T{}''(\cdot) \quad \forall t: t_{qj} \leq t \leq t_{ej}$$

More specific results

- Three cases:
  - Possibly asymmetric, linear delay function ($\chi = 1$): A&N
  - Possibly asymmetric, non-linear delay function ($\chi > 1$): N
  - Symmetric, non-linear delay function (χ free): A&N

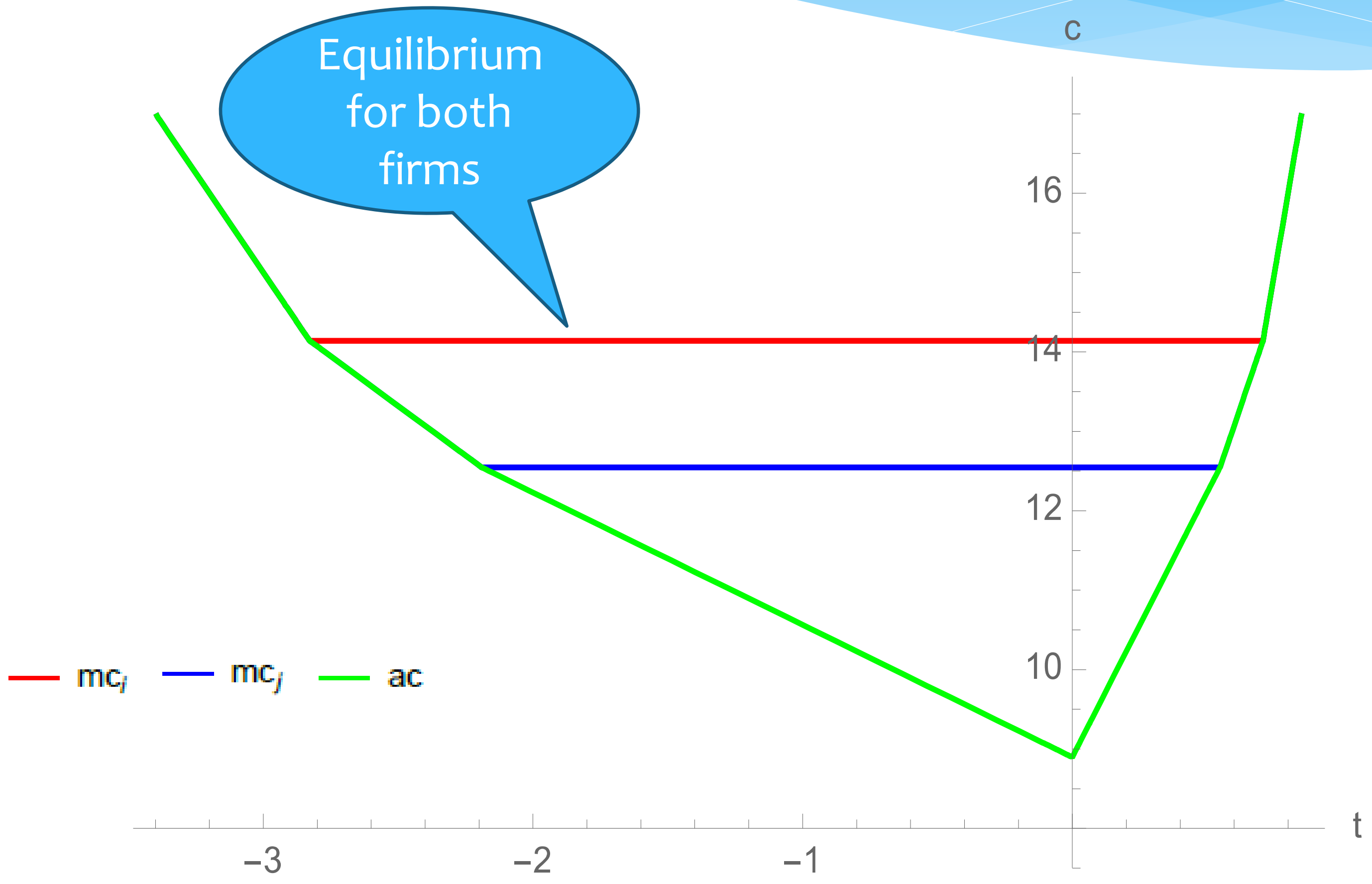
Asymmetric, linear delay ($\chi = 1$)

- Ratios of aggregate growth rates
  - Atomistic 1
  - Two firms, one active: 1/2
  - Two firms, both active: 2/3
  - Optimum 1/2
- Time intervals

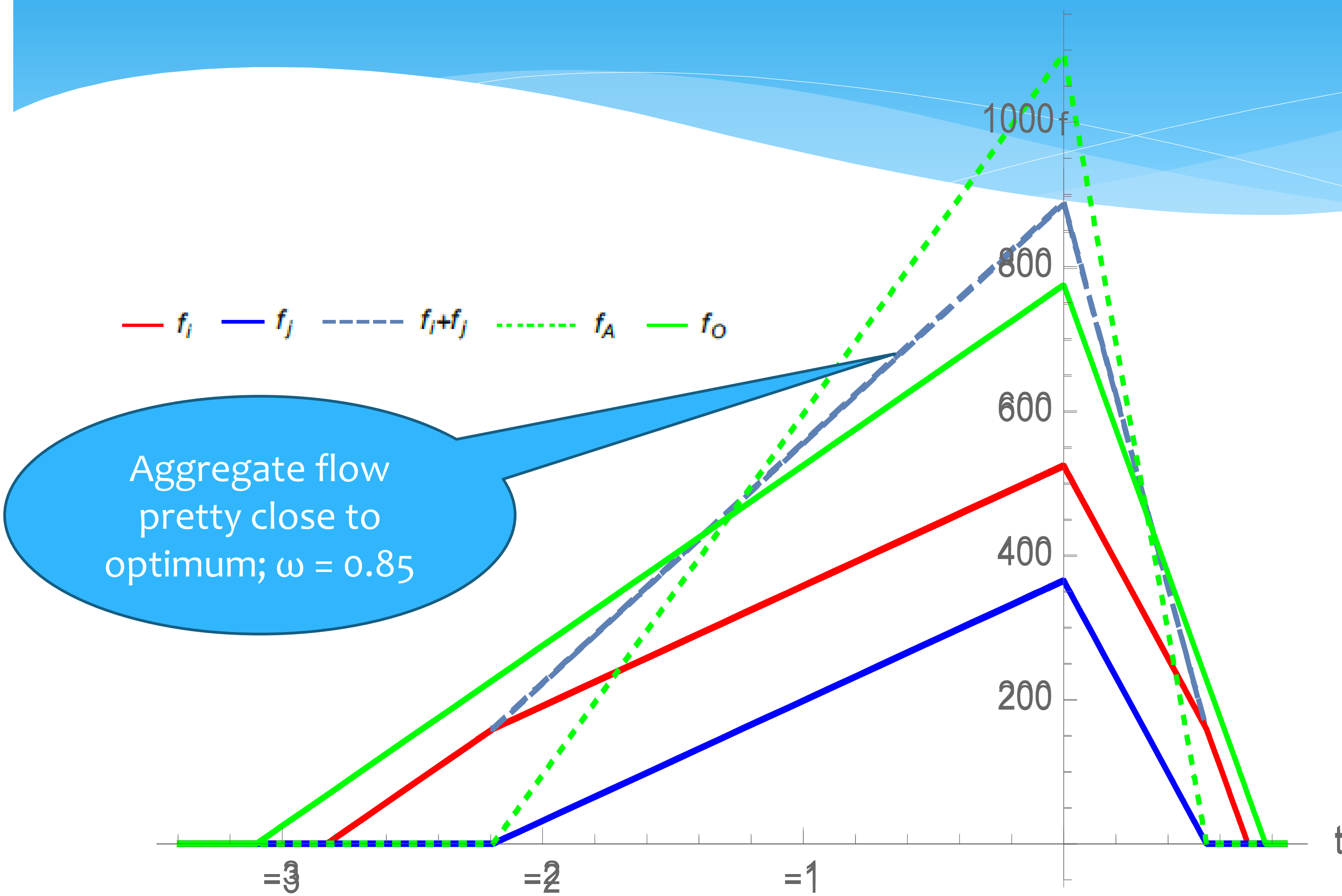
$$\begin{cases} t_{qi} = -\dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\gamma}\cdot\sqrt{2\cdot N_i + N_j}}{\sqrt{\beta}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{qj} = -\dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\gamma}\cdot\sqrt{3\cdot N_j}}{\sqrt{\beta}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{ej} = \dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\beta}\cdot\sqrt{3\cdot N_j}}{\sqrt{\gamma}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{ei} = \dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\beta}\cdot\sqrt{2\cdot N_i + N_j}}{\sqrt{\gamma}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \end{cases}$$

$$\begin{cases} t_{qO} = -\dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\gamma}\cdot\sqrt{2\cdot N_i + 2\cdot N_j}}{\sqrt{\beta}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{qA} = -\dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\gamma}\cdot\sqrt{N_i + N_j}}{\sqrt{\beta}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{eA} = \dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\beta}\cdot\sqrt{N_i + N_j}}{\sqrt{\gamma}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \\ t_{eO} = \dfrac{\sqrt{2}\cdot\sqrt{\alpha}\cdot\sqrt{\beta}\cdot\sqrt{2\cdot N_i + 2\cdot N_j}}{\sqrt{\gamma}\cdot\sqrt{\beta+\gamma}\cdot\sqrt{K}} \end{cases}$$

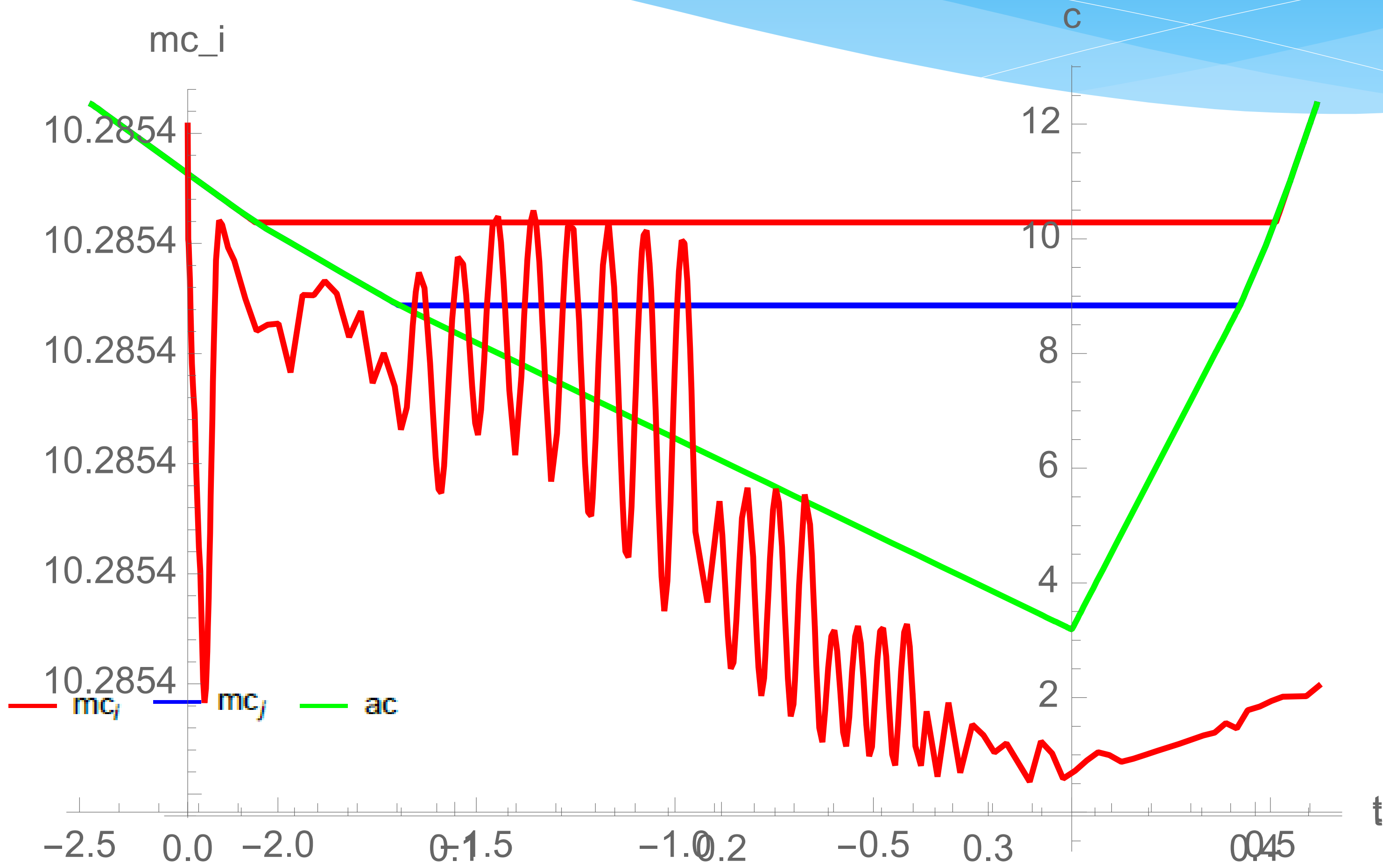
Numerical: costs



Numerical: flows

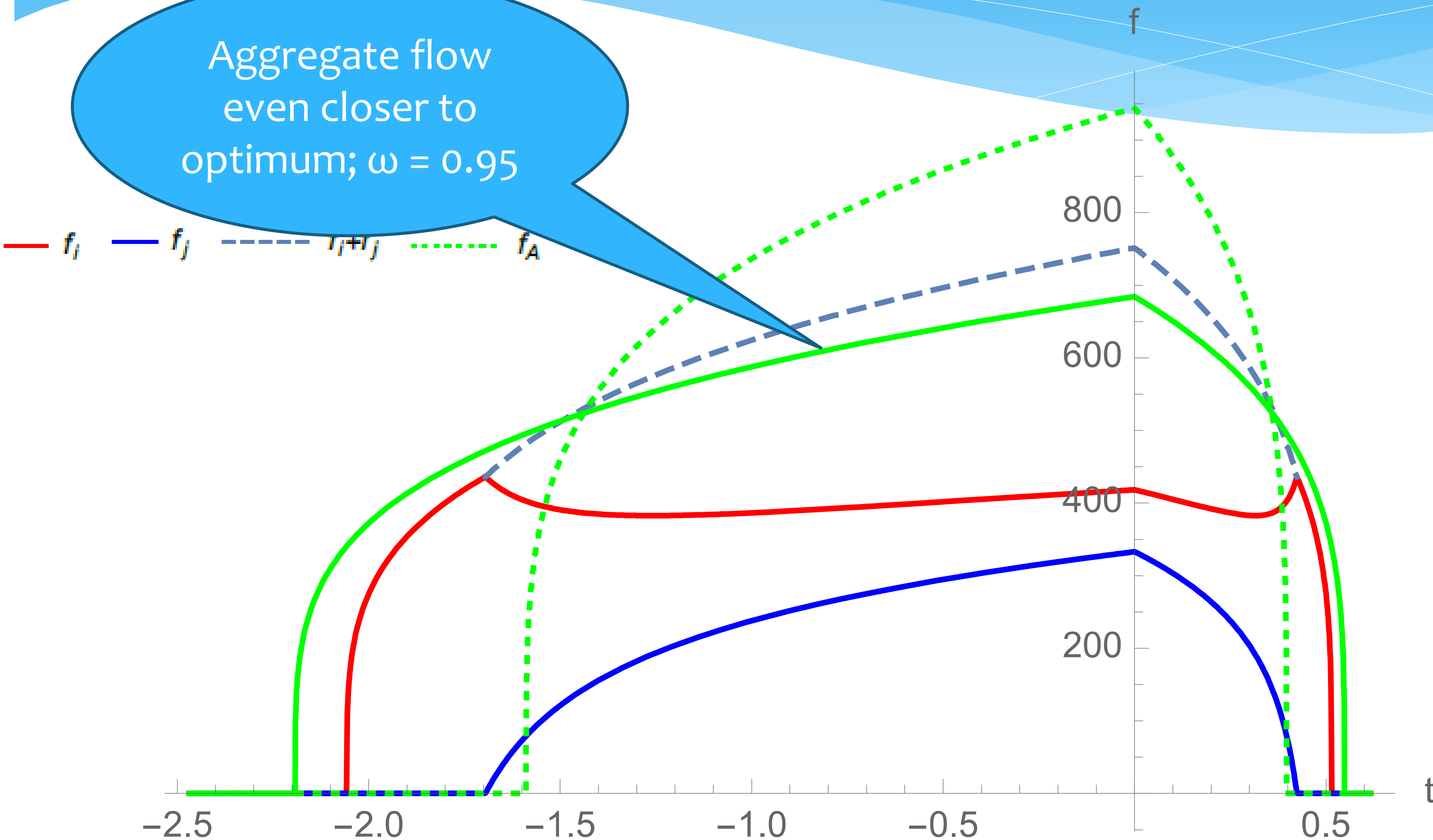


Asymmetric, non-linear delay ($\chi = 4$)



Flows

Aggregate flow even closer to optimum; ω = 0.95



Symmetric, non-linear delay (χ free)

$$f_S(t) = \begin{cases} \left(\frac{K}{2} \cdot \left(\frac{1}{1+\frac{1}{2}\cdot\chi} \cdot \frac{\beta}{\alpha} \cdot \; t - t_{qS} \; \right) \right)^{\frac{1}{\chi}} & \forall t: \; t_{qS} \le t \le 0 \\ \left(\frac{K}{2} \cdot \left(\frac{1}{1+\frac{1}{2}\cdot\chi} \cdot \frac{\gamma}{\alpha} \cdot \; t_{eS} - t \; \right) \right)^{\frac{1}{\chi}} & \forall t: 0 < t \le \mathrm{t}_{eS} \end{cases}$$

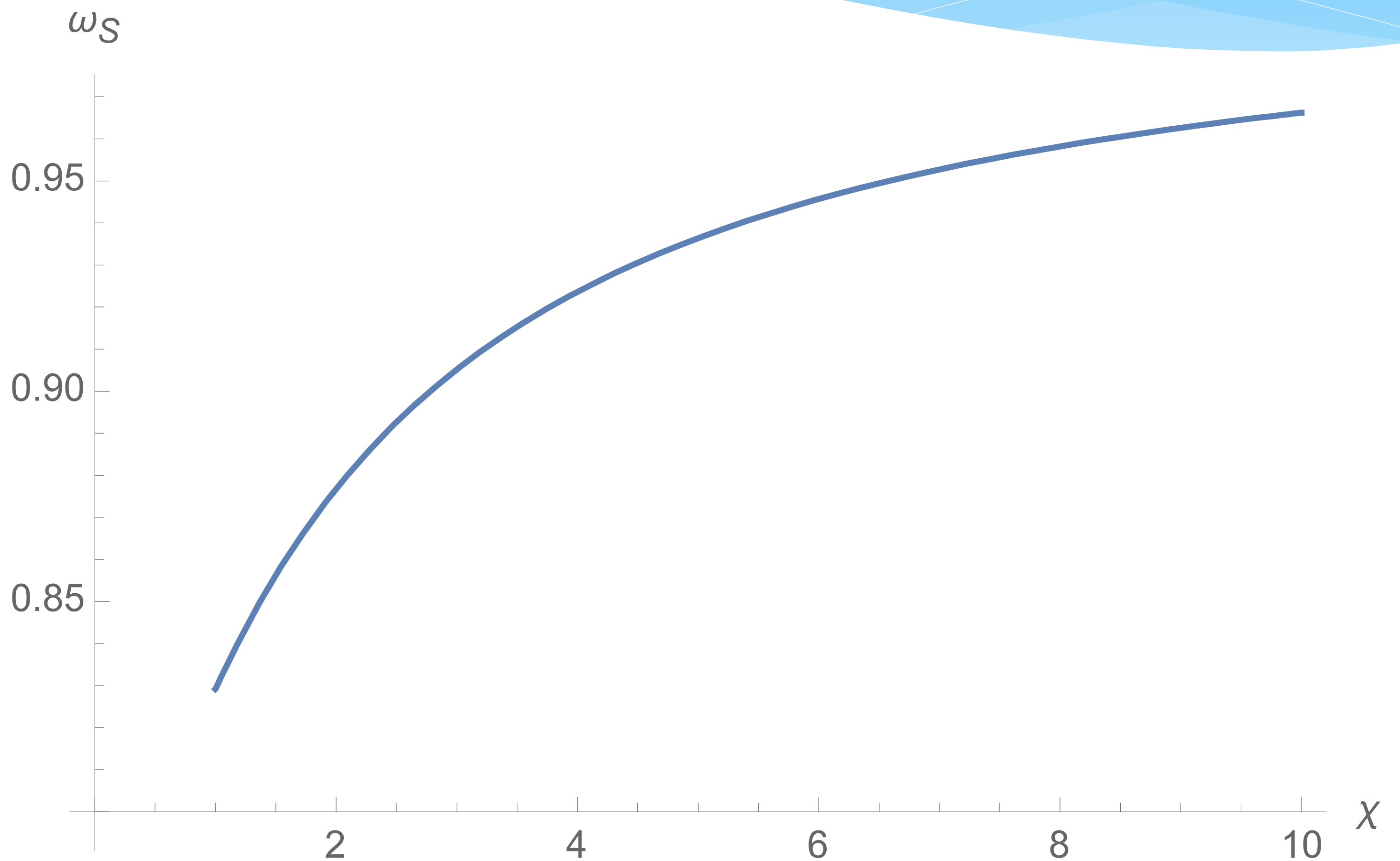
$$t_{qS} = - \; 1+\tfrac{1}{2}\cdot\chi^{\;\frac{1}{1+\chi}} \cdot \Psi \cdot \frac{\alpha}{\beta}$$

$$t_{eS} = \; 1+\tfrac{1}{2}\cdot\chi^{\;\frac{1}{1+\chi}} \cdot \Psi \cdot \frac{\alpha}{\gamma}$$

Factors ½ would be:
1 for optimum
0 for atomistic

Relative efficiency

Function of χ alone



Conclusions

- Dynamic equilibrium exists for $\alpha < \gamma$
- With two firms, pretty efficient ($\omega > 0.85$)
- Quite some follow-up questions
  - Agnew congestion technology
  - Leader – fringe, Bertrand
  - Heterogeneity
  - Stochastic delays