The Structure of the RELU-TRAN L.A.-Parking Model

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1. Introduction

The purpose of this technical report is to explain the theoretical structure of **RELU-TRAN L.A.-Parking** (from now on referred to as **RELU-TRAN LA-P**) and to show how this model extends the original RELU-TRAN model described in Anas and Liu (2007). [1]

The RELU-TRAN LA-P model is a spatially detailed computer general equilibrium model of a regional economy that includes components reflecting the key choices of consumers, producers, landlords and developers and their interactions in the labor, housing, outputs for industries and land markets.

The goal of RELU-TRAN LA-P is to make seven significant extensions to RELU-TRAN, so that the power and accuracy of this model in examining certain policies, in particular transportation-related fringe benefits such as employer paid parking, in the Greater Los Angeles Region can be examined. These extensions include: a) adding parking costs into the consumers generalized transportation costs; b) adding road traffic congestion for trips that occur within geographic zones, not just between two different geographic zones; c) allow vehicle ownership decision; d) allow producers to subsidize employees parking expenditures associated with commuting by car to the workplace as well consumers' shopping parking expenditures; e) allow for carpooling as an alternative transportation mode to the workplace, f) allow for parking as another building type that can be produced and thus, model parking supply and g) have a more realistic representation of the government sector. In particular the model incorporates a wide array of tax instruments, including corporate income tax, property taxes, sales taxes, payroll taxes and taxes on individual labor and capital income. The main activities of the government sector are transferring incomes, purchase goods and services and raising revenue through taxes.

This report is structured as follows. Section 2 describes how the Greater Los Angeles Region is represented in the RELU-TRAN LA-P model. Section 3 provides a detailed description of the theoretical structure of the model and section 4 presents the functional forms used in the simulation for utility and

[1] Relu-Tran was developed under a grant from the United States National Science Foundation to Dr. Alex Anas. It is a CGE model calibrated and tested for the Chicago MSA. The RELU-TRAN2 is an extension of RELU-TRAN that includes gasoline consumption and the choice of vehicle type and it is also calibrated for the Chicago MSA. The Relu-Tran L.A.-P was developed under the LA-Plan project which is funded through the University of California's Multi-campus Research Program and Initiative (MRPI). The original Relu-Tran L.A. model is calibrated and tested for the Greater Los Angeles Region but it does not account for parking or car ownership.

production functions. Section 5 discusses employer-paid parking as a tax-exempt fringe benefit and why employees may still choose to cash in lieu of the parking. Finally, the last section offers conclusions and possible extensions.

2. Representing the Los Angeles MSA

The Los Angeles Region in the RELU-TRAN LA-P model is based on the Southern California Association of Governments (SCAG) region and thus, it is composed of six counties: LA County, Ventura County, San Bernardino County, Riverside County, Orange County and Imperial County. According to the Census, the Los Angeles Metropolitan Area consists of Los Angeles and Orange Counties. The Inland Empire is a metropolitan area located east of the Los Angeles Metropolitan Area and consists of Riverside County and San Bernardino County. Imperial County is a relatively small, predominantly agricultural county, located about 200 miles southeast of Los Angeles.

In the RELU-TRAN LA-P model, the Greater Los Angeles Region is represented by a system of 97 model zones and by an aggregation of the major and local road networks. At each zone, a homogeneous and fixed land area is available for the development of residents and non-residential uses. In addition, each zone has a housing market, a labor market, and output markets for industries. These markets are competitive.

All intra-zonal trips use a congestible local road that is an aggregation of the underlying street and minor road system. Trips choose a path over the road-links which are an aggregation of major roads and highways. Each one-way inter-zonal link is represented by a capacity used to calculate equilibrium flow congestion which determines equilibrium monetary and time costs on the link.

The urban economy is partly closed in the sense that the total population of consumers in the urban area is fixed and exogenously given. Yet, some share of the urban production will not be consumed in the urban area, but will be exported in exchange for some monetary expenditures for goods and services produced or owned (for example land owned by absentee landowners) outside the urban economy but consumed by urban residents. Therefore, there is no interurban migration, utility levels of consumers are endogenously determined and model allocates the given aggregate population of consumers among the 97 zones only. Table 1 presents the distribution of the model zones across the six counties.

Table 1: RELU-TRAN LA-P Model Zones

Model Zones	County
Zones 1 through 46	Los Angeles
Zones 47 through 49	Ventura
Zones 50 through 66	Orange
Zones 67 through 80	San Bernardino
Zones 81 through 95	Riverside
Zones 96 through 97	Imperial

3. Theoretical Model Structure

Economic Agents: Consumers, Producers, Real Estate Investors and Government

The Relu-Tran L.A.-P is microeconomic in structure and consists of consumers, producers, real estate investors and the government that sets taxes and decides how the tax revenues generated by various taxes are redistributed in the economy.

Consumers and firms are competitive in all markets and take prices as given. Consumers do not pay parking costs at their residence locations. However, while consumers must pay for parking when they shop, consumers who commute by car to work receive a parking fringe benefit. The amount of the fringe benefit provided by an employer to any employee and which may be excluded from gross income does not exceed the cap established by the IRC.[2] It is further assumed that a consumer parks in the zone where he resides, works, or shops and that firms lease an amount of parking spaces that reflects employee demand for parking. All industry types within a zone of the county lease parking spaces at the same exogenous market parking price for the zone.

Car traffic is assumed to generate congestion, but public transit neither suffers from, nor contributes to congestion.

Government's Tax Instruments

The government's tax instruments include a proportional personal income tax on the labor income received by employed consumers and on the income received by unemployed consumers, a personal income tax on capital income (interests and capital gains) as well as a payroll tax rate on the wage paid by employers to their employees. Both payroll tax rates and personal income tax rates vary based on the consumer's earnings. In addition, the government also levies a retail sales tax and charges property taxes. Under a retail sales tax, the price that the consumer has to pay is assumed to rise by the amount of the tax. Sales between producers are exempt from taxation. Sales taxes are assumed to vary by county. Property taxes are modeled as an ad-valorem tax on building values (including land value).

Industry Disaggregation

The RELU LA-P model has nine basic industries ($r = 1,...,9$) : agriculture (1), finance/insurance/real estate (2), manufacturing (3), public administration (4), services (5), transportation warehousing (6), retail (7), wholesale (8) and utility (electric, gas, sanitary services) (9). Each industry can produce in every

[2] There are no data on how many employees in southern California actually receive employer-paid parking subsidies greater than the cap on tax-exempt parking subsidies or on the total value of parking subsidies that are subject to taxation. Therefore, to facilitate the numerical implementation of the model further in the paper we assume that the parking subsidies offered do not exceed IRC caps.

model zone and can import inputs in any input group (except buildings) from all other zones. All industries are assumed to be competitive.

All variants of a good (1)-(9) are used as intermediate inputs in the production of other goods in the region. Outputs from basic industries $r = 1,\ldots,9$ can also be exported from the region. In addition, each basic industry uses business capital, space in commercial, industrial, public and parking (garages) buildings and labor from each of the skills groups (income quartiles) of the employed consumers.

There are also two specialized industries for each building type. One provides construction and the other demolition of that type of building. Since each of the 97 model zones is the potential site of two housing types (where $k = 1$ is single-family type and $k = 2$ is multi-family type buildings) and four business building types (with $k = 3$ is commercial/office, $k = 4$ is industrial, $k = 5$ is public buildings and $k = 6$ is parking garages) there are 2*(2+4)=12 such industries. These specialized industries also use the primary inputs as well as inputs from all basic industries and sell their services (construction and demolition) only to building developers.

3.1 Consumer's Utility Maximization Problem

The consumer's utility maximization problem in the RELU-TRAN LA-P can be described by a three-level nested optimization setup. In the third (last) nest, the consumer knows his disposable income after having made the decision to own a car or not and after having decided by what mode to commute to work every day. This disposable income must be allocated among the housing and composite good quantities purchased on shopping trips by each of the available modes. In the second nest, the consumer chooses a mode for the commute given the prior car ownership decision. Finally, in the outer first nest whether to own a car or not is decided. Next we provide further details on the consumer choice process.

Consumer's Residence-Work-Housing-Car Choice

Consumers are divided into four skill levels ($f = 1,2,3,4$) in the labor market that correspond to quartiles of the income distribution in the model calibration. N_f represents the exogenous number of consumers in skill group f .

Each consumer decides whether he will be employed or unemployed (voluntary unemployment), in which zone to work if employed and how many hours of labor to supply in the work zone. In addition, all consumers also choose a zone of residence and one of the housing types that are available in their zone of residence. Consumers also decide how many shopping trips to make from their place of residence to each shopping zone and how much to shop of retail goods there. Another decision households make is regarding automobile ownership.

Therefore, all consumers in RELU (both employed and unemployed) choose among discrete bundles (i, j, k, c) where $i = 1,...,97$ residential zones, $j = 0,...,97$ job zones, $k = 1,2$ housing types (single-family, $k = 1$; multi-family housing, $k = 2$) and $c = 0,1$ number of cars owned. It should be mentioned that $j = 0$ represents a fictitious nonspatial work zone to capture the voluntary choice of unemployment.

Continuous variables, conditional on each discrete bundle are the housing floor space for (i, j, k, c), labor supplied (measured in hours) at zone $j > 0$, shopping trips from zone i to all retail zones $z = 1,...,97$, and the quantity of retailed goods to buy at each zone z.

Retailed goods in different zones are imperfect substitutes and all zones are patronized as the consumer's utility exhibits an extreme taste for retail varieties in the sense that a consumer will shop in each retail zone no matter how high the price of the retail goods offered at that location. Travel time is valued at the wage.

For a given residence-work-housing-car ownership type (i, j, k, c), each consumer of skill/income level f maximizes his utility with respect to the annual amount of retailed goods quantities in each industry r to purchase at each potential retail destination, z (Z_{rz}), the number of hours of labor supply annually (H_{ijf}) and the amount of housing floor space of type k to rent (b_{ik}) taking into account the following constraints:

i) budget constraint

Consumers are myopic and spend all their income of each period during that period. Total monetary expenditures on retail goods (so $r = 7$), commuting, annual costs of car ownership (if a car owner, that is, $c = 1$) and housing space cannot exceed the household's net-of income tax wage after travel time for commuting and shopping plus government income transfers (GT).

Total consumption expenditures for a consumer of skill f are represented by:[3]

$$\sum_{z=1}^{97}[p_{rz}(1+t_z^{sales}) + c_{ijf}(g_{izc} + \Delta_j(1-v_f)w_{jf}G_{izc})]Z_{rz} + b_{ik}R_{ik} + Oc \qquad (1)$$

where, the prices of the retail goods are the mill prices at the retail location plus the monetary cost of the travel from residence to the retail zone.

[3] Since industries are competitive, the consumer is indifferent about industry of employment because all industries within the same work zone offer the same wage. However, wages differ by work zone (j) and by household skill type (f).

Δ_j is a parameter that takes the value of 1 if the consumer is employed and the value of 0 if the consumer is unemployed. Thus, if the consumer chooses employment, then $j > 0$ and $\Delta_j = 1$.

O is the annualized cost of car ownership for those who choose to be car owners. It is multiplied by $c = 0$ and drops out, if a consumer does not own a car.

p_{rz} is the unit mill price of retail goods sold in the shopping zone z for the retail industry r

t_z^{sales} is the retail sales tax on purchased consumer goods from the retail industry at shopping zone z

c_{ijf} is the fixed and exogenous coefficients that measure the number of retail trips necessary to purchase one unit of a retail good

v_f is the unit income tax rate for an employed consumer with income/skill f

w_{jf} is the hourly wage paid to a consumer with skill f employed in a job zone j

g_{izc} is the two-way mode and route composite monetary expected travel cost from residence i to shopping zone z under car ownership c (from TRAN)

G_{izc} is the two-way mode and route composite expected travel time of one round shopping trip from i to z for any consumer under car ownership c (from TRAN)

R_{ik} is the rent of residential floor space of type k in the residential zone i

On the other hand, **total income** is represented as

$$\Delta_j (1 - v_f) w_{jf} H_{ijcf} + (1 - \Delta_j v_f - (1 - \Delta_j) v_f^u) M_f - \Delta_j d g_{ijc} + GT_{ijkcf} \tag{2}$$

where,

GT_{ijkcf} represents the government transfer to a consumer of skill-level f with residence in zone i in a housing type k and car ownership c and working in zone j;

d is the exogenous number of commutes (work days) in a year (assumed to be the same for all employed consumer skill types);

g_{ijc} is the two-way mode and route composite monetary expected travel cost from residence i to job zone j under car ownership c (from TRAN);

H_{ijcf} is the annual labor supply (measured in hours) in the work zone j of a consumer of skill-type f and car ownership c with residence at i;

v_f^u is the unit income tax rate for an unemployed consumer (applies to nonwage income only) with skill type f, with $v_f^u < v_f$;

M_f represents the nonwage income of the consumer with skill type f and is specified as follows

$$M_f = \frac{\xi_f \left[\sum_{k=0}^{5} \sum_{i=1}^{97} \frac{\rho}{1+\rho} V_{ik} S_{ik} + \Theta \right]}{N_f} \tag{3}$$

where $1 < \xi_f < 0$ is the aggregate share of skill group f in the aggregate regional nonwage income and ρ is the real interest rate. V_{ik} is the stationary state market price (per unit of floor space or land) of a type-k real estate asset (building) in zone i where a $k = 0$ denotes vacant land as an asset and $k > 0$ denotes each of the other building types in the model. S_{ik} is the stationary real estate stock of a type-k real estate asset (building) in zone i.

According to equation (3) each consumer within his type receives an equally share of the aggregate share of skill group f in the aggregate regional nonwage income. The aggregate regional nonwage income has two components: the first component on the right hand side of (3) is the aggregate discounted annual return from real estate in the region (land and buildings) and the second component (Θ) is the aggregate asset income from all other sources inside and outside the region that is owned by consumers in the region.

ii) Time Constraint

$$\Delta_j \left[dG_{ijc} + H_{ijcf} \right] + \sum_{z=1}^{97} c_{ijf} Z_{rz} G_{izc} \leq H_f \tag{4}$$

According to (4) the total annual time spent commuting to work and working plus the total annual time spent retail shopping cannot exceed the total annual time endowment the consumer has. An unemployed consumer incurs no commuting travel time or cost but make shopping trips.

H_f represents the consumer with skill type f 's total annual time endowment for work and travel (measured in hours);

G_{ijc} is the two-way mode and route composite expected travel time over all available travel modes of one round trip from i to the workplace j for any consumer under car ownership c (from TRAN);

$Gizc$ is the two-way mode and route composite expected travel time over all available travel modes of one round shopping trip from i to k for any consumer under car ownership c (from TRAN).

From the consumer maximization problem we get the marshallian demands for each consumer and the annual labor supply. Each employed consumer with skill type f determines his annual labor supply (measured in hours) to the chosen labor market at j as follows

$$H_{ijcf} = H_f - dG_{ijc} - \sum_{z=1}^{97} c_{ijf} Z^*_{rz|ijkc} G_{izc} \geq 0 \tag{5}$$

since for an optimal voluntary choice of employment $\Delta_j = 1$, $\forall j > 0$ and $Z^*_{rz|ijkc}$ represents the marshallian demand of retail goods of the consumer which are the solution to the continuous choices in the consumer's utility maximization problem.

Substituting the marshallian demands of the consumer back into the utility function we get the optimized deterministic utility level $\tilde{U}_{ijkc|f}$ and finally the complete indirect random utility function

$$U^*_{ijkc|f} = \tilde{U}_{ijkc|f} + e_{ijkc|f} \tag{6}$$

where $e_{ijkc|f}$ are idiosyncratic taste constants and represent the stochastic part of the random utility function.

Given this optimization for each (i, j, k, c), each consumer of skill type f compares the complete location choice set (i, j, k, c) and chooses the most Since these idiosyncratic tastes are distributed among the households for each (i, j, k, c), choices are described probabilistically in the form of a discrete choice model. The probability that the consumer of skill type f chooses a specific location choice set (i, j, k, c) is

$$P^*_{ijkc|f} = \mathrm{Pr}ob\left[\tilde{U}_{ijkc|f} + e_{ijkc|f} > \tilde{U}_{stnm|f} + e_{abde|f} \quad \forall (s,t,n,m) \neq (i,j,k,c)\right] \tag{7}$$

where $P^*_{ijkc|f}$ is the probability that a randomly selected consumer prefers the choice set (i, j, k, c). Assuming that $e_{ijkc|f}$ is independently, identically Gumbel distributed with dispersion parameter λ_f, the choice probabilities are given by the following multinomial logit model:

$$P^*_{ijkc|f} = \frac{\exp(\lambda_f \tilde{U}_{ijkc|f})}{\sum_{s=1}^{97}\sum_{t=0}^{97}\sum_{n=1}^{2}\sum_{m=0}^{1} \exp(\lambda_f \tilde{U}_{stnm|f})} \text{ with } \sum_{i=1}^{97}\sum_{j=0}^{97}\sum_{k=1}^{2}\sum_{c=0}^{1} P^*_{ijkc|f} = 1 \tag{8}$$

Once consumers choose among the discrete states (i, j, k, c), including $j = 0$ and $c = 0$, they must choose the mode of travel for each trip and the routing of that trip over the road network if the mode is car. These choices are treated in the TRAN part of the LA-P CGE model. RELU connects with TRAN via the mode- and route-composite trip times and monetary costs, which are given by the matrices $[G_{ijc|f}]$ and $[g_{ijc|f}]$. The RELU-TRAN L.A.-P does not treat traffic congestion by time of the day, so all consumers who use the car as the transportation mode either to work or/and shopping experience the same congestion.

Consumer's Mode Choice

We consider five types of transportation modes (m) available for both commuting and shopping trips with $m=1$ (solo driver), 2 (carpool/vanpool), 3 (rail), 4 (bus) and 5 (other). The solo driver car mode is available only to car owners. If one does not own a car, then all trips, commutes as well as shopping trips, must be made either by carpool or by non-car modes. Note nevertheless that a consumer may own a car but not use it to commute to work.

In addition, the monetary travel cost for the auto mode includes two components: one accounting for the mode specific average variable travel costs per mile for a trip (shopping, commuting) from the residence zone i to the job zone j (shopping zone z) such as fuel costs plus other mode specific travel costs from the residence zone i to the job zone j (shopping zone z) such as parking costs.

The two-way monetary travel costs from i to j or z (g_{ijc} and g_{izc}) used by consumers to make choices in the RELU part would thus be given by:

$$g_{ijc} = \sum_{m=1c}^{5} \pi_{m|ijc}(c_{ijm} + c_{jim} + \hat{c}_{jm}) \text{ with } \hat{c}_{jm} = 0 \text{ for } m = 3,4,5 \quad \text{(residence-job)} \tag{9}$$

$$g_{izc} = \sum_{m=1c}^{5} \pi_{m|izc}(c_{izm} + c_{zim} + \hat{c}_{zm}) \text{ with } \hat{c}_{zm} = 0 \text{ for } m = 3,4,5 \quad \text{(residence-shop)} \tag{10}$$

with the subscript $c=0$ (if a car is not owned).

The two-way time travel costs from i to j or z (g_{ijc} and g_{izc}) used by consumers to make choices in the RELU part is given by:[4]

$$G_{ijc} = \sum_{m=1c}^{5} \pi_{m|ijc}(t_{ijm} + t_{jim}) \quad \text{(residence-job)} \tag{11}$$

$$G_{izc} = \sum_{m=1c}^{5} \pi_{m|izc}(t_{izm} + t_{zim}) \quad \text{(residence-shop)} \tag{12}$$

[4] Time spent cruising for parking is absent in the model.

with $c = 0$ (if a car is not owned) and where,

c_{ijm} is the one-way monetary cost (excluding parking costs) per consumer for mode m from zone i to zone j (for example: fuel costs or transit fare).

c_{jim} is the one-way monetary cost (excluding parking costs) per consumer for mode m from zone j to zone i. Note that if a consumer carpools then $c_{ij2} = \frac{c_{ij1}}{n}$, where n is the exogenous constant car occupancy rate in persons per vehicle. The larger the carpool, the lower the monetary cost per person.

$\hat{c}_{jm}$ is the out-of-vehicle costs (such as parking costs at work) of a trip using mode m to zone j.[5] This cost is job zone specific and it does not depend on the origin of the trip. If a consumer parks free at work or commutes by mode $m = 3,4,5$ then $\hat{c}_{jm} = 0$, otherwise $\hat{c}_{jm} > 0$. If a consumer carpools then $\hat{c}_{j2} = \frac{\hat{c}_{j1}}{n}$. It is interesting to note that passing the true cost of parking amenities to car commuters ($\hat{c}_{jm} > 0$) has therefore the potential to affect consumer choices related to vehicle ownership, vehicle trips and mode choice.

t_{ijm} is the exogenously one-way travel time from zone i to zone j with mode m. For mode $m = 2$ (carpool) we may assume that each additional person in the car adds another $\hat{t}$ (for example 5 minutes) to the travel time because collecting and distributing passengers makes the trip circuitous. Thus, $t_{ij2} = t_{ij1} + (n-1)\hat{t}$. Splitting the monetary cost of driving and parking is an incentive to carpool, while the added travel time is a disincentive.

t_{jim} is the one-way travel time from zone j to zone i with mode m, with $t_{ji2} = t_{ji1} + (n-1)\hat{t}$.

$\pi_{m|ijc}$ is the probability that a consumer chooses mode m for a trip originating in zone i and terminating in zone j conditional on car ownership c. This implies that it is assumed that a consumer will choose each available mode m with some probability. These mode choice probabilities can be computed by using a mode choice model in multinomial logit form:

[5] It should be noted that the only parking cost considered in the current version of the model is the total parking fee that a consumer may or may not pay at the workplace zone. Other factors such as the time spent searching and queuing for a parking space or the time spent walking to the final destination or the expected fine for illegal parking are not considered in the model.

$$\pi_{m|ijc} = \frac{\delta_{m|ijc} \exp \breve{\lambda}(C_{m|ijc} + h_{m|ijc})}{\sum_{m=1c}^{5} \delta_{n|ijc} \exp \breve{\lambda}(C_{n|ijc} + h_{n|ijc})} \quad \text{with} \sum_{m=1c}^{5} \pi_{m|ijc} = 1 \tag{13}$$

where the subscript $c = 0$ (if a car is not owned) and,

$C_{m|ij}$ is the expected round trip generalized cost of travel per consumer from zone i to zone j with mode m. The generalized cost of travel includes both the value of time and the monetary in- and out- of vehicle costs. The generalized cost for the auto modes ($m = 1,2$) takes into account the generalized cost of travel by auto (which does not include parking costs), obtained from the route-choice model. For further details on how to compute the generalized cost of travel see Anas and Liu (2007).

$h_{m|ij}$ denotes a mode specific constant

$\breve{\lambda}$ is the mode choice cost-dispersion coefficient

$\delta_{m|ij}$ is a variable that takes the value of 1 if transport mode m belongs to the set of transportation modes available to trips from i to j. This is true for at least one transportation mode; otherwise, if the transport mode is not available then the variable takes the value 0. The choice set for the study of employer paid parking is solo driver, carpool, rail, bus and other. However, if a consumer is not a car owner then the choice set is reduced to carpool, rail, bus and other.

Consumer's Route Choice

The consumer will also choose each available route r with some probability when commuting by car (whether solo driver or carpool). Consumers choose the route that minimizes their perceived generalized cost of travel. These route choice probabilities can be computed by using a route choice model in multinomial logit form:

$$\Pr ob_{r \in \phi_{ij}} = \frac{\exp(-\hat{\lambda} RGCOST_{r \in \phi_{ij}})}{\sum_{\forall s \in \phi_{ij}} \exp(-\hat{\lambda} RGCOST_{s \in \phi_{ij}})} \quad \text{with} \sum_{\forall s \in \phi_{ij}} \Pr ob_{r \in \phi_{ij}} = 1 \tag{14}$$

where,

$\hat{\lambda}$ is the route choice cost-dispersion coefficient

ϕ_{ij} is the set of permissible highway routes, each route being itself a set of sequential links on the road network connecting origin i with destination j

$RGCOST_{r \in \phi_{ij}}$ is the generalized costs (monetary & time value) for route r, which is determined as

$$RGCOST_{r \in \phi_{ij}} = \sum_{\forall l} a_{l \in r \in \phi_{ij}} \left[VOT * Time_l + MCOST_l \right] \quad (15)$$

where,

a_l is a variable that takes the value of 1 if the road-link l belongs to route r and a value of 0 otherwise

VOT is the value of time for travel on the network, assumed uniform across all commuters

$Time_l$ is the congested one-way travel time (in minutes) on the road-link l, which is determined as

$$Time_l = \alpha_l * length_l * \left[1 + \beta_l \left[\frac{Flow_l}{CAP_l} \right]^{\varphi} \right] \quad (16)$$

where,

α_l is the free-flow (uncongested) travel time per mile on the road-link l

$Flow_l$ is the vehicle traffic flow on the road-link l (determined endogenously in the highway network equilibrium. See Anas and Kim (1990) for further details.

CAP_l is the calibrated capacity of link l

$length_l$ is the length of the road-link l (measured in miles)

$MCOST_l$ is the one-way per commuter monetary cost on a link, assumed not to depend on congestion and determined as

$$MCOST_l = UCOST_l * length_l * \frac{1}{n} \quad (17)$$

where,

$UCOST_l$ is the exogenous unit monetary cost ($/mile/vehicle)

n is the constant car occupancy rate in persons per vehicle

Finally, we can determine the $C_{m|ij}$, that is, the expected round trip generalized costs of travel per consumer by mode m from zone i to zone j as

$$C_{1|ij} = -\frac{1}{\hat{\lambda}} \ln \sum_{s \in \phi_{ij}} \exp(-\hat{\lambda} * RGCOST_{s \in \phi_{ij}}) - \frac{1}{\hat{\lambda}} \ln \sum_{s \in \phi_{ji}} \exp(-\hat{\lambda} * RGCOST_{s \in \phi_{ji}}) + \hat{c}_{j1} \quad (18)$$

$$C_{2|ij} = -\frac{1}{\hat{\lambda}} \ln \sum_{s \in \phi_{ij}} \exp(-\hat{\lambda} * RGCOST_{s \in \phi_{ij}}) - \\ -\frac{1}{\hat{\lambda}} \ln \sum_{s \in \phi_{ji}} \exp(-\hat{\lambda} * RGCOST_{s \in \phi_{ji}}) + \hat{c}_{j2} + 2VOT * (n-1)\hat{t} \tag{19}$$

$$C_{m|ij} = VOT * (t_{ijm} + t_{jim}) + c_{ijm} + c_{jim} \text{ if } \delta_{m|ij} = 1 \text{ when } m = 3,4,5 \tag{20}$$

3.2. Producers Behavior

All firms producing in the same zone j and industry r are perfectly competitive profit maximizers in input and output markets, employing a constant-returns-to-scale production function that combines capital (K_r), floor space (B_{rk}), labor (L_{rmf}) and intermediate goods from (supplying) industries s (Y_{sn}) to produce a zone specific composite commodity (X_{rj}), charging the same price and paying the same wages and rents. The supplying industries in the model include only the basic industries, $s = 1,\ldots,9$. Firms are myopic profit maximizers and perfectly competitive. The number of firms is therefore indeterminate, and the model finds aggregate output, employment by zone j and industry r.

Let R_{j6} denote the long-run resource cost of parking to (receiving) industry r in zone j. Parking subsidies are per vehicle and not per commuter. In addition, there is no cashing out of parking costs and producers rent just enough parking floor space to meet their employees and shoppers demand for parking space.

Total expenditures for the composite good of industry r in zone j (except retail, $r = 7$) are given by:

$$E_{rj} = \rho K_r + \sum_{s=1}^{9} \sum_{n=1}^{97} \sum_{c=0}^{1} \left[p_{sn} + \sigma_s g_{njc} \right] Y_{sn} + \sum_{k=0}^{5} R_{jk} B_{rk} + \\ + \sum_{m=1}^{5} \sum_{f=1}^{4} (w_{jf} - (R_{j6} - \hat{c}_{j1}))(1 + t_f^{payroll}) L_{rmf} + \sum_{f=1}^{4} d(R_{j6} - \hat{c}_{j1}) \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right] \tag{21}$$

for $r = 1,\ldots,6,8,\ldots,21$ where,

g_{njc} is the monetary cost of a consumer commuting from residence n to zone j;

σ_s is a factor that converts transport cost g_{njc} to the monetary cost of freight transport per unit of industry s output;

ρ is the exogenous price of business capital (i.e, the real interest rate);

p_{sn} is the price of the output sold at the place of production.[6]

Note that the delivered price of the same output purchased by other producers located at some zone j is given by $p_{sn} + \sigma_s g_{nj}$.

Sales between producers are exempt from taxation. However, producers must pay payroll taxes. Therefore, the wage paid by producers consist of a nominal wage net any fringe benefits ($w_{jf} - (R_{j6} - \hat{c}_{j1})$), actually paid to the employed consumer, and a payroll tax, that is, $(w_{jf} - (R_{j6} - \hat{c}_{j1}))(1 + t_f^{payroll})$, with $t_f^{payroll}$ the producer (employer) payroll tax rate when the consumer (employee) earning is $w_{jf} - (R_{j6} - \hat{c}_{j1})$.

In addition, producers also subsidize employees parking expenditures associated with commuting by car to the workplace. $d(R_{j6} - \hat{c}_{j1})$ is the annual parking subsidy per vehicle paid by industry r in zone j. It is calculated as the product of the exogenous number of days per year for which a commute is required and the parking subsidy per day. The parking subsidy per day is calculated by the gap between the long run resource cost of parking to the firm (R_{j6}) and the (possibly zero) contribution of the car commuter for parking at work ($\hat{c}_{j1}$).

$\sum_{f=1}^{4} \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right]$ represents the number of vehicles per day subsidized through employer- paid parking. $\frac{L_{r1f}}{8d}$ represents the number of vehicles commuting to zone j by consumers of skill type f who work in industry r and drive alone; $\frac{L_{r2f}}{8dn}$ represents the number of vehicles commuting to zone j by consumers of skill type f who work in industry r and carpool.

It is further assumed that all employed consumers commuting by car are able to find a parking space in the zone where they work. Therefore, total annual demand for parking spaces (measured in terms of the number of vehicles that need to be parked per year) in zone j (excluding parking demand from the retail industry) is given by:

$$\sum_{r=1}^{6} \sum_{f=1}^{4} d \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right] + \sum_{r=8}^{21} \sum_{f=1}^{4} d \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right] \quad (22)$$

[6] Freight congestion costs related to the flow of intermediate goods between zones are ignored in the current version of the Relu-Tran LA-P model.

Let $\bar{l}$ represent the exogenous amount of floor space occupied by one vehicle. Total annual demand of parking floor space demanded by all firms in zone j (excluding retail) is thus given by

$$\sum_{r=1}^{6} B_{r6} + \sum_{r=8}^{21} B_{r6} \tag{23}$$

where $B_{r6} = \bar{l} d \sum_{f=1}^{4} \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right]$ represents the annual demand for floor-space in building garages by industry r (except retail) .

Inserting $B_{r6} = \bar{l} d \sum_{f=1}^{4} \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right]$ back into (21) yields total expenditures for the composite good of a particular industry r (except retail) in job zone j as

$$\begin{aligned} E_{rj} = \rho K_r &+ \sum_{s=1}^{9} \sum_{n=1}^{97} \sum_{c=0}^{1} \left[p_{sn} + \sigma_s g_{njc} \right] Y_{sn} + \sum_{k=0}^{5} R_{jk} B_{rk} + \\ &+ \sum_{m=1}^{5} \sum_{f=1}^{4} (w_{jf} - (R_{j6} - \hat{c}_{fj}))(1 + t_f^{payroll}) L_{rmf} + (R_{j6} - \hat{c}_{j1}) \frac{B_{r6}}{\bar{l}} \end{aligned} \tag{24}$$

where $B_{r6} = \bar{l} d \sum_{f=1}^{4} \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right]$ must also be taken into account.

Total expenditures for the composite good of the retail industry $r = 7$ ***in zone*** j ***are given by:***

$$\begin{aligned} E_{rj} = \rho K_r &+ \sum_{s=1}^{9} \sum_{n=1}^{97} \sum_{c=0}^{1} \left[p_{sn} + \sigma_s g_{njc} \right] Y_{sn} + \sum_{k=0}^{5} R_{jk} B_{rk} + \\ &+ \sum_{m=1}^{5} \sum_{f=1}^{4} (w_{jf} - (R_{j6} - \hat{c}_{j1}))(1 + t_f^{payroll}) L_{rmf} + \\ &+ \sum_{f=1}^{4} (R_{j6} - \hat{c}_{j1}) d \left[\frac{L_{r1f}}{8d} + \frac{L_{r2f}}{8dn} \right] + \\ &+ \sum_{i=1}^{97} \sum_{k=1}^{2} \sum_{f=1}^{4} \left[(R_{j6} - \hat{c}_{j1}) \left[\frac{N_{ijkfc=0} \pi_{m=2|ijkfc=0} + N_{ijkfc=1} \pi_{m=2|ijkfc=1}}{n} + N_{ijkfc=1} \pi_{m=1|ijkfc=1} \right] c_{ijf} Z_{rjf} + \right] \end{aligned}$$

(25)

In the case of the retail industry, producers also provide a parking subsidy to its customers (shoppers). The additional total subsidy that the retail industry in zone j (which is also a shopping area) provides to its shoppers is given by:

$$\sum_{i=1}^{97}\sum_{k=1}^{2}\sum_{f=1}^{4}\left[(R_{j6}-\hat{c}_{j1})\left[\frac{N_{ijkfc=0}\pi_{m=2|ijkfc=0}+N_{ijkfc=1}\pi_{m=2|ijkfc=1}}{n}+N_{ijkfc=1}\pi_{m=1|ijkfc=1}\right]c_{ijf}Z_{rjf}\right] \quad (26)$$

In this case the retail industry annual demand for parking floor space is given by:

$$B_{r6}=\bar{l}d\sum_{f=1}^{4}\left[\frac{L_{r1f}}{8d}+\frac{L_{r2f}}{8dn}\right]+$$
$$+\bar{l}\sum_{i=1}^{97}\sum_{k=1}^{2}\sum_{f=1}^{4}\left[\frac{N_{ijkfc=0}\pi_{m=2|ijkfc=0}+N_{ijkfc=1}\pi_{m=2|ijkfc=1}}{n}+N_{ijkfc=1}\pi_{m=1|ijkfc=1}\right]c_{ijf}Z_{rjf}$$

(27)

Inserting (27) back into (25) yields total expenditures for the composite good of a retail industry in zone j as

$$\begin{aligned}E_{rj}=\rho K_r &+\sum_{s=1}^{9}\sum_{n=1}^{97}\sum_{c=0}^{1}\left[p_{sn}+\sigma_s g_{njc}\right]Y_{sn}+\sum_{k=0}^{5}R_{jk}B_{rk}+\\ &+\sum_{m=1}^{5}\sum_{f=1}^{4}(w_{jf}-(R_{j6}-\hat{c}_{fj}))(1+t_f^{payroll})L_{rmf}+\\ &+(R_{j6}-\hat{c}_{j1})d\sum_{f=1}^{4}\left[\frac{L_{r1f}}{8d}+\frac{L_{r2f}}{8dn}\right]+\\ &+(R_{j6}-\hat{c}_{j1})\left[\frac{B_{r6}}{\bar{l}}-d\sum_{f=1}^{4}\left[\frac{L_{r1f}}{8d}+\frac{L_{r2f}}{8dn}\right]\right]\end{aligned} \quad (28)$$

Producers choose the amount of capital, labor supply (measured in hours), building floor space and intermediate inputs in order to minimize total expenditures to produce a target amount of the industry good. From this minimization problem we get the conditional input demand functions of the industry r producing in zone j. Substituting back the conditional input demands into the expenditure functions yields industry r cost function as:

$$C_{rj}(w_{jf}\forall f;\rho;R_{jk}\forall k\setminus 6;p_{sn}+\sigma_s g_{nj};d;(R_{j6}-\hat{c}_{j1});X_{rj};t_f\forall f) \quad (29)$$

The zone specific commodity prices are then determined from the zero profit condition, since free entry in each zone ensures that profit maximizing firms make zero economic profit in the competitive market. Hence, the condition that price equals marginal (and average) cost yields

$$p_{rj} = \frac{\partial C(\cdot, X)}{\partial X} = \frac{C(\cdot, X_{rj})}{X_{rj}} \tag{30}$$

Note that even if a consumer does not pay directly for parking ($\hat{c}_{j1} = 0$ and $\hat{c}_{z1} = 0$), parking costs are bundled into the price of intermediate and retail goods. Moreover, a commuter can also pay for his/her parking costs in the form of reduced wage compensation.

3.3 Real Estate Investors

The real estate investors' model is developed as a stationary-state or long-run equilibrium model. Real estate investors buy, rent (or keep vacant), convert, and sell property (buildings and vacant land). There are no transaction costs with buying and selling real estate properties. In contrast to producers and consumers, who are myopic, real estate investors operate with perfect foresight and are risk-neutral profit maximizers.

Real estate investors also pay income taxes. The tax base is the rent net of financial maintenance costs plus capital gains net of financial construction (demolition) costs minus property tax payments. Let $v^{investor}$ represent the investor's income tax rate. The property tax in zone i is an ad valorem tax on property value at the rate $t_{ik}^{property}$. It is further assumed that nonfinancial costs (common and idiosyncratic) are excluded from tax calculations.[7]

Time is modeled in discrete periods of one year in duration. Rents are received at the beginning of a period, maintenance and nonfinancial construction (demolition) costs are incurred also at that same period while all other costs and property taxes are paid at the end of a period. The income tax is paid at the moment revenue is received.

To the extent that that current property taxes are paid before the property is converted to another property type (that is, construction of a building type-k occurs on vacant land or a building type-k gets demolished), then the property tax is a sunk cost in the construction and demolishing decisions. As such it does not affect the investor's construction and demolition choice probabilities. Future property taxes affect nevertheless indirectly these decisions through the capitalization into future property values of the new property types.

[7] Nonfinancial costs reflect the income equivalents of nuisances, know-how value, value of time, and other nonfinancial issues related with property investment. Examples include bad tenants, monitoring the work of an inefficient builder or searching for information about construction and/or demolition technologies. One implication of assuming that nonfinancial costs are not deductible in computing the income tax is that the income tax is not a neutral tax.

Investors as landlords

An investor that acts as a landlord operates floor space in residential ($k=1,2$), commercial buildings ($k=3,4,5$) and parking garages ($k=6$) by maximizing profit under perfect competition and decides whether to offer a unit amount of floor space for rent or withhold it from the rental market. In addition, this investor also decides whether to rent or keep vacant a unit amount of land ($k=0$). As such, each investor at each point in time, must maximize profits by comparing the revenue obtained under occupancy

$$(1-v^{investor})\left(R_{ik}-D_{iko}\right)+d_{iko} \quad \text{for } k=0,\ldots,6 \tag{31}$$

and the revenue accruing under vacancy

$$(1-v^{investor})\left(-D_{ikv}\right)+d_{ikv} \quad \text{for } k=0,\ldots,6 \tag{32}$$

where D_{iko} and D_{ikv} represent respectively, common maintenance costs to all landlords of type-k properties on a unit floor space (or a unit amount of land) in zone i if the unit is occupied by a tenant or is vacant. Terms d_{iko} and d_{ikv} are i.i.d. Gumbel idiosyncratic maintenance costs that vary across these landlords with dispersion parameter ϕ_{ik}.

Let q_{ik} denote the probability that a landlord operating a type-k unit (apartment, parking or land) in zone i will decide to offer it for rent. From the binominal logit calculus, this probability is given by

$$q_{ik}(R_{ik})=\frac{\exp[\phi_{ik}(1-v^{investor})(R_{ik}-D_{iko})]}{\exp[\phi_{ik}(1-v^{investor})(R_{ik}-D_{iko})]+\exp[\phi_{ik}(1-v^{investor})(-D_{ikv})]} \tag{33}$$

while 1-q_{ik} is the probability that the unit floor space will remain vacant.

At the beginning of a time period, before idiosyncratic uncertainty is resolved, landlords do not know if they will rent or keep a unit vacant. Hence, the expected annual profit of a landlord operating a type-k unit floor space in zone i before the idiosyncratic vacancy costs are revealed is given by:

$$\omega_{ik}(R_{ik})=\frac{1}{\phi_{ik}}\ln[\exp[\phi_{ik}(1-v^{investor})(R_{ik}-D_{iko})]+\exp[\phi_{ik}(1-v^{investor})(-D_{ikv})]],\ k=1,\ldots,6 \tag{34}$$

The rental price on vacant land, R_{i0}, is taken as exogenous.

Investors as Developers

Investors who behave as developers buy the services of specialized industries that construct or demolish buildings. These investors are profit maximizing and competitive firms taking building asset prices, unit construction and demolition prices as given. Developers determine how much of a given amount of land remains vacant or if a particular type of building is build on it. We assume that the structural densities of all building types are exogenous.

Following Anas and Liu (2007) we assume that there is a one-period lag for any construction (or demolition) to occur. Developers buy vacant land (or a building) in the beginning of the period, then they operate the asset for rental during the period (act as landlords) and finally, by the end of the period, they decide on whether and what kind of building to built (or whether to demolish an existing building).

At the beginning of a time period, before idiosyncratic uncertainty is resolved, developers do not know if they will build a type-k building on a unit of vacant land and if they will demolish an existing type-k building.

Hence, the expected economic profit for an investor in land in zone i is given by the sum of the expected capital gains discounted to the beginning of the time period net of financial costs minus interest payments (no down payment is assumed) plus the after-tax rent collected on vacant land at the start of the period net the year-end property taxes paid at the ad valorem tax rate $t_{i0}^{property}$ plus the investor's discounted income tax rebate:

$$E[\max(\Pi_{i00}, \Pi_{i0k}; k=1,\ldots,5)] + (1 - v^{investor})R_{i0} - \frac{[t_{i0}^{property} + r]}{1+\rho}V_{i0} + \\ + \frac{v^{investor}\left[1 + t_{i0}^{property} + r\right]}{1+\rho}V_{i0} + \frac{\rho}{1+\rho}V_{i0} \tag{35}$$

where

r is the before-tax discount rate, $v^{\text{int}erest}$ is the personal tax rate on interest income (bonds) and $\rho = (1 - v^{\text{int}erest})r$ represents the after-tax discount rate;

Π_{i00} are the net of tax capital gains discounted to the beginning of the time period from keeping vacant a unit amount of land in zone i;

Π_{i0k} are the net of tax capital gains discounted to the beginning of the time from constructing a unit amount of floor space in a type-k building in zone i.

The expectation $E[\bullet]$ is calculated as

$$E[\bullet] = \frac{\ln\left[\exp[\frac{\Phi_{i0}(1-v^{investor})V_{i0}}{1+\rho}] + \sum_{k=1}^{5}\exp\Phi_{i0}[\frac{(1-v^{investor})(V_{ik} - p_{R+k,i})m_k}{1+\rho} - C_{i0k}]\right]}{\Phi_{i0}} - V_{i0} \tag{36}$$

with

V_{i0} is the stationary state market price per unit of land in zone i;

V_{ik} is the stationary state market price per unit of floor space of a type-k building in zone i;

Φ_{i0} is the dispersion parameter of i.i.d. Gumbel distributed idiosyncratic costs associated with keeping land vacant or constructing on it;

$p_{R+k,i}$ represents construction (industry) prices per unit floor space;

m_k is the exogenous structural density (square feet of floor space per acre of lot size) of building type-k;

$\frac{1}{m_k}$ is acres of land used up for each square foot of floor space of type-k that is constructed;

C_{i0k} is the unit per acre construction nonfinancial cost in zone i;

$Ci00$ is the unit per acre nonfinancial cost in zone i for keeping land undeveloped and is assumed to be zero.

The probability that a profit maximizing developer with a unit amount of vacant land builds a type-k building on it ($k > 0$) is given by[8]

$$Q_{i0k}(V_{i0}, V_{i1}, ..., V_{i5}) = \frac{\exp\Phi_{i0}[\frac{(1-v^{investor})(V_{ik} - p_{R+k,i})m_k}{1+\rho} - C_{i0k}]}{\exp[\frac{\Phi_{i0}(1-v^{investor})V_{i0}}{1+\rho}] + \sum_{s=1}^{5}\exp\Phi_{i0}[\frac{(1-v^{investor})(V_{is} - p_{R+s,i})m_s}{1+\rho} - C_{i0s}]}$$

(37)

Finally, in competitive asset market equilibrium, the price of each asset is bid up until the rate of return generated by that asset becomes equal to the rate of return of the financial investment. Thus, the competitive bid for vacant land in zone i is determined by setting (35) equal to zero and then solving this equation for the year-beginning asset bid price for land as follows:

$$V_{i0} = \frac{(1+\rho)(1-v^{investor})}{(1+t_{i0}^{property} + r)(1-v^{investor}) + \rho} R_{i0} + \\ + \frac{\ln\left[\exp[\frac{\Phi_{i0}(1-v^{investor})V_{i0}}{1+\rho}] + \sum_{k=1}^{5}\exp\Phi_{i0}[\frac{(1-v^{investor})(V_{ik} - p_{R+k,i})m_k}{1+\rho} - C_{i0k}]\right]}{\Phi_{i0}\left[(1+t_{i0}^{property} + r)(1-v^{investor}) + \rho\right]/(1+\rho)} \quad (38)$$

On the other hand, the expected economic profit for an investor in type-k buildings in zone i is represented as

[8] This probability is determined from stochastic profit maximization and it expresses $\Pr ob[\Pi_{i0k} > \Pi_{i00}]$, for $k = 1,...,6$.

$$E[\max(\Pi_{ikk},\Pi_{ik0})]+\omega_{ik}(R_{ik})-\frac{1}{1+\rho}[t_{ik}^{property}+r]V_{ik}+\frac{v^{investor}V_{ik}}{1+\rho}\left[1+t_{ik}^{property}+r\right]+\frac{\rho}{1+\rho}V_{ik}$$

(39)

where,

Π_{ikk} represents the net of tax capital gains from keeping as is a unit amount of floor space in a type-k building in zone i;

Π_{ik0} represents the net of tax capital gains of demolishing a unit amount of type-k floor space in a building in zone i.

The expectation $E[\bullet]$ in (39) is calculated as

$$E[\bullet]=\frac{\ln\left[\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}(\frac{V_{i0}}{m_k}-p_{R+K+k,i})-C_{ik0}]+\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}V_{ik}-C_{ikk}]\right]}{\Phi_{ik}}-V_{ik}$$

(40)

where,

Φ_{ik} is the dispersion parameter of i.i.d. Gumbel distributed idiosyncratic costs associated with keeping a building as is or demolishing it, $p_{R+K+k,i}$ are the demolition (industry) prices per unit of floor space and C_{ikk} represent nonfinancial costs with keeping the type-k building unchanged.

The probability that a developer in zone i will demolish a unit floor space in a type-k building is given by

$$Q_{ik0}(V_{i0},V_{ik})=\frac{\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}(\frac{V_{i0}}{m_k}-p_{R+K+k,i})-C_{ik0}]}{\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}(\frac{V_{i0}}{m_k}-p_{R+K+k,i})-C_{ik0}]+\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}V_{ik}-C_{ikk}]}$$

(41)

The gross of tax start-of-the year asset bid price in zone i for $k=1,\dots,6$ is given by

$$\frac{(1+t_{i0}^{property}+r)(1-v^{investor})+\rho}{1+\rho}V_{ik}=\omega_{ik}(R_{ik})+$$
$$+\frac{\ln\left[\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}(\frac{V_{i0}}{m_k}-p_{R+K+k,i})-C_{ik0}]+\exp\Phi_{ik}[\frac{(1-v^{investor})}{1+\rho}V_{ik}-C_{ikk}]\right]}{\Phi_{ik}}. \quad (42)$$

3.4. Government

A single government sector approximates government activities at the federal, state and local levels. The main activities of the government sector are purchasing goods and services (both durable and nondurable), transferring incomes, and raising revenue through taxes.[9] We assume that the government's discount rate is the gross-of-tax interest rate. The government's annual expenditures (*AGE*) divides into the aggregate public administration costs and nominal transfers to consumers:

$$\sum_{j=0}^{97} C_{rj} + \sum_{f=1}^{4}\sum_{i=1}^{97}\sum_{j=0}^{97}\sum_{k=1}^{2}\sum_{c=0}^{1} N_{ijkcf}\, GT_{ijkcf} \tag{43}$$

where C_{rj} is determined by (21) and $r=4$ to represent the public administration cost function. Note that government transfers can be heterogeneous. In particular, the government may transfer its net revenue back to consumers in proportion to their labor income, e,g, through changing their payroll tax rate.

The government's annual net revenue (AGR) is given by the tax revenue from all sources less disbursements.

$$
\begin{aligned}
AGB = & \sum_{r=1}^{19}\sum_{f=1}^{4}\sum_{i=1}^{97}\sum_{j=0}^{97}\sum_{k=1}^{2}\sum_{c=0}^{1} N_{ijkcf}\, P_{ijkc|f} \left[\Delta_j v_f w_{rjf} (H_f - dG_{ij} - \sum_{r=9}^{9}\sum_{z=1}^{97} c_{ijf} Z_{rzf} G_{iz}) + (\Delta_j v_f + (1-\Delta_j) v_f^u) M_f \right] + \\
+ & \sum_{r=9}^{9}\sum_{z=1}^{97}\sum_{f=1}^{4}\sum_{i=1}^{97}\sum_{j=0}^{97}\sum_{k=1}^{2}\sum_{c=0}^{1} N_{ijkcf}\, P_{ijkc|f}\, Z_{rz|ijcf}\, p_{rz} t_z^{sales} + \\
+ & \sum_{r=1}^{19}\sum_{f=1}^{4}\sum_{i=1}^{97}\sum_{j=1}^{97}\sum_{k=1}^{2}\sum_{c=0}^{1} N_{ijkcf}\, P_{ijkc|f} \left[H_f - dG_{ij} - \sum_{r=9}^{9}\sum_{z=1}^{97} c_{ijf} Z_{rzcf} G_{iz} \right] w_{rjf} t_f^{payroll} + \\
+ & \sum_{i=1}^{97}\sum_{k=0}^{6} S_{ik} \left[q_{ko}(R_{ik} - D_{iko}) - q_{kv} D_{ikv} \right] v^{investor} + \\
+ & \sum_{i=1}^{97}\sum_{k=0}^{6} S_{ik} \sum_{k=0}^{5} \frac{Q_{i0k}}{1+r} \left[(V_{ik} - p_{R+k,i}) m_k - (1 + r + t_0^{property}) V_{i0} \right] v^{investor} + \\
+ & \sum_{i=1}^{97}\sum_{k=0}^{6} S_{ik} \sum_{k=0}^{5} \frac{Q_{ik0}}{1+r} \left[(\frac{V_{i0}}{m_k} - p_{R+K+k,i}) - (1 + r + t_k^{property}) V_{ik} \right] v^{investor} + \\
+ & \sum_{i=1}^{97}\sum_{k=0}^{6} S_{ik} \sum_{k=0}^{5} \frac{Q_{i0k} V_{ik} + (1 - Q_{ik0}) V_{ik}}{1+r} t^{property}
\end{aligned}
$$

(44)

---

[9] Government can also raise revenue by issuing tax-exempt bonds at a nominal rate lower than current investments. However, we abstract from this revenue source in this version of the Relu-Tran LA model.

It is further assumed that each year the government budget constraint must balance and the overall government expenditure is exogenous but increases at a constant rate, g . This implies that government spending captured by (44) must follow the same path. To meet this budget equilibrium constraint requirement, we accompany a particular parking/transportation tax with a reduction in another tax, either on a lump-sum basis or through reductions in marginal tax rates.

3.5 Equilibrium Conditions

In addition to the utility and profit maximization conditions and the government budget constraint, several other conditions are necessary to close the model. At general equilibrium, the factor markets for land, building floor space and labor as well as the market for the locally produced composite commodities must clear in each zone i. Furthermore, firms (producers) in each zone i must make zero economic profits. Solving all the equilibrium conditions determines in each zone, the rental price (per square foot) of each type of floor space, the hourly wage for each skill level and the output price for each industry.

Next we focus on the additional equilibrium conditions that must be taken into account when parking is endogenously determined in the RELU-TRAN model. For further details on the remaining equilibrium conditions see Anas and Liu (2007).

In the particular case where parking is also a building type produced in the model ($k=6$) we have that demand for parking floor space in zone j must equal its supply in that zone:

$$\sum_{r=1}^{21} B_{k|rj} = S_{jk} q_{jk}(R_{jk}) \text{ with subscript } k=6. \tag{45}$$

Moreover, in stationary equilibrium, for each type of building, the flow of demolished floor space equals that constructed so that the stock of each building type in each model zone remains stable in each time period. Therefore, with parking buildings the following condition must also be satisfied:

$$S_{jk} Q_{jk0}(V_{j0}, V_{ik}) = m_k S_{j0} Q_{j0k}(V_{j0}, V_{j1}, ..., V_{j6}) \text{ with subscript } k=6. \tag{46}$$

In each model zone, the total amount of land, J_j, is given. Hence the acres taken up by each real asset type including land that remains vacant, must add up to J_j. This requires that for each model zone j, the following land condition is also met:

$$\sum_{k=0}^{6} \frac{S_{jk}}{m_k} = J_j, \quad j=1,...,97. \tag{47}$$

4. Functional forms in the Simulation

Next we specify the functional forms for consumers' utility function and producers' production function currently in use in the RELU-TRAN LA-P model.

4.1. Utility Function

$$U_{ijkc|f} = \alpha_{ikc|f} \ln\left[\sum_{\forall z} \phi_{z|ijkc} Z_{z|ijkcf}^{\eta_f}\right]^{\frac{1}{\eta_f}} + \beta_{ikc|f} \ln b_{ijkc|f} + \gamma_f G_{ij} + E_{ijkc|f} + e_{ijkc|f} \quad (48)$$

with $k = 1,2$, $c = 0,1$, $\alpha_f + \beta_f = 1$, $-\infty < \eta_f < 1$ and $\phi_{z|ijf} \geq 0$ and where,[10]

$\alpha_{ikc|f}$ is the share of the consumer's disposable income for a consumer of skill-level f who lives in model zone i and chooses housing type k and car ownership c spent on the retailed goods purchased from all the zones in the model;

$\beta_{ikc|f}$ is the share of the consumer's disposable income for a consumer of skill-level f who lives in model zone i and chooses housing type k and car ownership c spent on housing floor space rented in type k housing in zone i;

$\eta_f = \frac{s-1}{s}$ where $s = \frac{1}{1-\eta_f}$ is the elasticity of substitution among the retailed goods;

$\phi_{z|ijkc}$ are constants representing the inherent attractiveness of the retail location z for consumers conditional on (i, j, k, c). If $\phi_{z|ijkc} = 0$, the retail zone is not attractive for the consumer and excluded from the shopping travel.

$E_{ijkc|f}$ is a constant taste-effect for the combination of the discrete choices (i, j, k, c). This constant has the same value for all consumers of skill-type f with discrete choice (i, j, k, c). These terms vary across different skill-types to capture the idea that their tastes are vertically differentiated due to systematic (unobserved) effects that are not captured in the rest of the utility function.

[10] When $\eta_f = 1$, the sub-utility is linear (substitution between goods is perfectly elastic); when $0 < \eta_f < 1$, substitution between goods is elastic; when $\eta_f = 0$, sub-utility is Cobb-Douglas (unit elasticity of substitution); when $\eta_f < 0$, the elasticity of substitution is inelastic; when η_f goes to negative infinity, we have Leontief sub-utility, goods are not possible to substitute.

$e_{ijkc|f}$ is the idiosyncratic utility constant. In contrast to $E_{ijkc|f}$, this term varies among consumers of the same skill-type f for a given (i, j, k, c). This implies that these terms capture horizontal taste differences across consumers of the same skill-type facing the same choices.

4.2. Production Function

Firms in industry r in zone j produce output X_{rj} with Cobb-Douglas production functions:

$$X_{rj} = A_{rj} K^{v_r} \left[\sum_{f=0}^{4} \kappa_{f|rj} L_f^{\theta_r} \right]^{\frac{\delta_r}{\theta_r}} \left[\sum_{k=0}^{5} \chi_{k|rj} B_k^{\zeta_r} \right]^{\frac{\mu_r}{\zeta_r}} \prod_{s=1}^{9} \left[\sum_{n=0}^{97} \upsilon_{sn|rj} Y_{sn}^{\varepsilon_{sr}} \right]^{\frac{\gamma_{sr}}{\varepsilon_{sr}}} \tag{49}$$

where $r = 1,...,21$, $v_r + \delta_r + \mu_r + \sum_{s=1}^{8} \gamma_{sr} = 1$, $k = 0$ represents vacant land and K, L_f, B_k and Y_{sn} are business capital, labor, buildings (floor space) and intermediate inputs from the other industries.

Note that while there are 21 receiving industries (r), there are only 9 supplying industries (s) which correspond to the basic industries. Both the construction and demolition industries buy from the basic industries, but not the other way around. The retail industry sells its products to both consumers and to basic and specialized industries and the construction-demolition industries sell their output to developers;

A_{rj} is a constant calibrated by (receiving) industry r and by location to account for place specific Hicksian-neutral productivity effects;

$v_r, \delta_r, \mu_r, \gamma_{sr} > 0$ are the cost shares of each of the input goods;

$\frac{1}{1-\theta_r} > 1$ is the elasticity of substitution between labor-skill types;

$\frac{1}{1-\zeta_r} > 1$ is the elasticity of substitution between building types;

$\frac{1}{1-\varepsilon_{sr}} > 1$ is the elasticity of substitution between inputs from the (supplying) industry s;

$\kappa_{f|rj}, \chi_{k|rj}, \upsilon_{sn|rj} \geq 0$ are terms that allow to specify input-specific bias as well as excluding some inputs from being used in production. For example, residential buildings can be excluded as an input good in production by setting $\chi_{k|rj} = 0$ for $k = 1,2$.

5. Employer-Paid Parking Further Discussion

In this section we discuss employer-paid parking as a tax-exempt fringe benefit and why employees may still choose to cash in lieu of the parking.

5.1. What are the tax incentives of offer free parking?

Let v_f represent the marginal tax rate on earned income for an employed consumer with income/skill f. This marginal tax rate is the sum of the federal marginal income tax rate, the state marginal income tax rate plus the social security and Medicare tax rate. The employee income is his wage income plus the taxable value of his fringe benefit. Let $t_f^{payroll}$ represent the employer payroll tax rate. We assume further that the employee is a solo driver and that each month an employer may lease parking spaces to accommodate his employee monthly parking needs.

Suppose that the employer in model zone j pays $\frac{d\$R_{j6}}{12}$/month to provide his employee a free parking space at work. Let $\frac{d\$R_{j6}}{12}$ also denote the monthly market leasing price of a parking space.

If the employee pays nothing to park, the employee receives a parking subsidy in the amount $\frac{d\$R_{j6}}{12}$/month. If instead the employee pays $\frac{d\$\hat{c}_{1j}}{12}$/month to park, the monthly parking subsidy is $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}$, that is, the difference between the out-of-pocket amount paid by the employer to secure the availability of the employee parking space and the price charged to the employee for use of that space.

This parking subsidy takes, nevertheless, the form of a nonwage compensation tax-exempt benefit. In fact, employer-paid parking is a tax-exempt fringe benefit that an employee may qualify for only by driving to work. This fringe benefit exists regardless of skill type (income) or residence location.

For illustration purposes let's assume also that the employee actually does not contribute anything for his parking expenditures at the workplace, $\frac{d\$\hat{c}_{1j}}{12} = 0$. Now suppose that the employer offers his employee a taxable $\frac{d\$R_{j6}}{12}$ payment a month. Then, the after-tax cash this employee receives is

$\frac{d\$R_{j6}}{12}(1-v_f)$ a month. Converting $\frac{d\$R_{j6}}{12}$/month of taxable salary into a tax-exempt parking subsidy of $\frac{d\$R_{j6}}{12}$/month saves employees commuting to work by car $\frac{d\$R_{j6}}{12} * v_f$ a month.

On the other hand, because this fringe benefit allows the employer to pay a lower salary, the employer also saves $\frac{d\$R_{j6}}{12} t_f^{payroll}$ a month. This tax exemption for employer-paid parking is thus an incentive for an employer to offer free parking at work and thus to subsidize driving to work.

Note that no tax deduction is allowed if the employee pays for parking. This means that if a commuter pays for parking at work, the Internal Revenue Code does not allow this cost to be deducted as work-related expenses. This asymmetry of the tax exemption for employer paid (but not employee-paid) parking may help explain why 95% of all automobile commuters in the United States park free at work. Yet, because employee's total compensation is divided between cash wages and fringe benefits, a higher fringe benefit in general comes at the expense of a lower wage for all employees and not just for those who drive to work.

For the sake of illustration, let's assume that labor productivity is independent of the travel mode used for commuting, payroll taxes are zero and there is no cashing out for parking spaces not used. If the firm only hires public transit users, then the wage reflects the marginal product of labor. However, if the firm hires both types of commuters, then the wage paid to both types of employees reflects the marginal product of labor net the parking subsidy. Therefore, while the trade-off between free parking and lower wage maybe appealing for solo drivers, it penalizes those employees who do not drive to work and thus get no parking fringe benefit.

It should be noted that the US Internal Revenue Code (IRC) also establishes caps on the amount of the parking fringe benefit that can be deductible. For example, in 1993 the tax exemption for employer-paid parking was capped at $155 a month and the employer-paid vanpool or transit subsidies were also exempt up to $60 a month. Both tax exemptions are nevertheless indexed to the cost of living. So, in 2004 the IRC exempted the first $195 a month of employer-paid parking subsidies from income taxation. Any subsidies above this value were treated as taxable income.

5.2. Why would an employee prefer after-tax cash to a free parking space?

Let the monthly parking subsidy be given by $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}$ and that the employer offers the employee a taxable $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}$ payment in lieu of that tax-exempt benefit. In addition, let's assume that the employee values the parking space only $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}(1-\varphi)$ a month, where $0 < \varphi < 1$. That is, the parking subsidy is worth less to the employee than it costs the employer. Then, provided that $\varphi \geq v_t$, the commuter will choose to receive the taxable amount of $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}$ since it would still be better off in $\frac{d(\$R_{j6} - \$\hat{c}_{1j})}{12}(\varphi - v_t)$ than with free parking.

6. Conclusions and Extensions

A word is in order regarding the potential effects of parking costs on consumers' choices in the RELU-TRAN LA-P. The current model can be used to investigate the effect of parking costs on a consumer's choice of travel mode. However, because households do not have different preferences over alternative parking types and alternative parking supply locations within a zone are not explicitly modeled, the current version of the RELU-TRAN LA-P model cannot be used to investigate the effects that parking costs have on the choice between different types of parking (-street parking, off-street surface parking lot, multi-storey facility or illegal parking) or different parking locations. Given these caveats, the current version of the RELU-TRAN LA-P model is suitable to examine the long-run effects of parking costs on consumers' residence location, job location or car ownership. The model is also suitable to examine how parking costs affect mode choices to work and shopping, vehicle miles travelled and transit ridership. The model is not suitable to examine short-term decisions involving trip length (parking cruise) or trip start time. Yet, parking choice behavior can be incorporated in the current version of the model by using a multinomial logit model, capable of accommodating random heterogeneity in consumer's tastes for parking type.

With the continuous development of mass rapid transit systems to provide accessibility to large city centers and reducing auto trips, park and ride can also be thought as an alternative transportation mode that is most likely to be used by those who shift from auto to transit. The current version of the RELU-TRAN LA-P does not include this type of transport mode. To the extent that a park-ride mode would have features that belong to more than one nest, a cross-nested logit model can be developed in future work to accommodate the possibility of park and ride.

Finally, a central empirical application of the RELU-TRAN L.A.-P model will be the analysis of current and alternative commuter parking policies and congestion fees on traffic congestion and carbon emissions in the Greater Los Angeles Region. This requires a detailed representation of the behavior of the public sector and the possibility of alternative revenue-recycling schemes.

So far all empirical applications of the RELU-TRAN model and its extensions to the analysis of alternative road-pricing policies (such as congestion taxes, gas taxes or a cordon tolling) to reduce traffic congestion and carbon emissions in a particular MSA have disregarded the importance of revenue recycling. However, a key issue is what to do with the revenues that are raised.

While the current version of the RELU-TRAN L.A.-P takes already into account the role of revenue recycling, it only considers one revenue-recycling scheme: the finance of government transfer payments. To the extent that alternative recycling schemes may have different economic, environmental and distributional impacts, it is important to consider more alternatives for revenue recycling when examining road-pricing or the reform of commuter parking subsidies.

A growing body of the analysis in environmental economics on the effectiveness of alternative environmental policies for environmental protection (see for example Goulder et al. 1999 or Parry et al. 1999) has shown that the welfare effects of new regulations (namely pollution taxes and auctioned pollution permits) can critically depend on how these policies interact with pre-existing tax distortions in the labor market and on how the revenue raised is used in the economy. Therefore, alternatively to lump-sum transfers, revenues can be used to improve economic efficiency by reducing the rates of distortionary taxes in the economy. Revenues can also be earmarked to subsidize public transit fares.

Within the transportation literature, De Borger and Wuyts (2009) have already shown that the existence of employer-paid parking raises the welfare gain of a budgetary-neutral increase in congestion taxes, independent of the policy instrument used to recycle the tax revenues. The reason is that congestion taxes not only correct congestion externalities, they also reduce the inefficiency caused by employer-paid parking. Another finding of this study concerns the relative welfare effects of different recycling instruments. In particular, De Borger and Wuyts (2009) show that, compared to earlier results reported in the literature, recycling higher congestion tax revenues to subsidize public transit may have more favorable welfare implications than to recycle revenues via reductions in labor taxes. The reason is that labor tax reductions and public transport subsidies have very different effects on congestion and on the demand for parking. While this study offers interesting insights on the interactions of transportation policies with pre-existing distortions outside the transportation sector, it is does not provide effects within a general equilibrium framework.

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