

Identifying Employment Subcenters The Method of Exponentially Declining Cutoffs

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Abstract

The standard method of identifying employment subcenters is due to Giuliano and Small (2001). While simple, robust, and easy to apply, the GS method has weaknesses. A major weakness is that, because it uses absolute employment density and employment cutoffs, it identifies “too few” subcenters at the metropolitan periphery. This paper presents a modification to the GS method aimed at remedying this deficiency. The modification entails using cutoffs that decline exponentially with distance from the metropolitan center, thereby giving consideration to the employment density of a location relative to that of its locality. The paper applies this method of exponentially declining cutoffs to the Los Angeles metropolitan area, and argues that the method succeeds in remedying the deficiency. Since the rate at which employment density falls off with distance varies substantially across metropolitan areas, the method is more valuable in identifying employment subcenters within a metropolitan area than in comparing the pattern of subcenters across metropolitan areas.

Keywords: subcenter, employment subcenter, subcenter identification, Giuliano-Small, Los Angeles

Identifying Employment Subcenters:

The Method of Exponentially Declining Cutoffs¹

It is standard to use the Giuliano-Small method to identify employment subcenters (Giuliano and Small, 1991). A subcenter is defined to be a set of contiguous² zones, each of which has an employment density of at least D employees/unit area and which together have a total employment of at least E . We refer to D as the employment density cutoff and to E as the total employment cutoff. And we denote this method of identifying subcenters as $GS(D,e)$, where $e = \frac{E}{1000}$; thus, for example, $GS(20,20)$ denotes the Giuliano-Small method when $D = 20$ employees/acre and $e = 20$ (so that $E = 20000$ employees) are used as the cutoffs. Figure 1 displays a map of the employment subcenters in the Los Angeles metropolitan area³ identified by applying $GS(20,20)$ to traffic analysis zones (TAZs) in 2003. The data we used in this paper is employment by TAZ of Los Angeles metropolitan area for year 2003. It is provided by Southern California Association of Governments and can be downloaded from <http://vcpa.ucr.edu/Data.html>.

One noteworthy feature, on which this paper focuses, is that, according to $GS(20,20)$, in 2003 there were no subcenters in Riverside, San Bernardino or Ventura Counties, even though the counties had 2000 Census populations of 1,545,387, 1,709,434 and 753,197, respectively. From the $GS(20,20)$ perspective, these three counties appear to be a vast, undifferentiated wasteland. Though this may be the perception of Los Angelenos, residents of each of the

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²The word “contiguous” is not without ambiguity. Two standard types of contiguity are rook contiguity and queen contiguity. Two zones are rook contiguous if they share a common border of finite length. Two zones are queen contiguous if they share only a common point. Giuliano and Small (1991) defined two zones to be contiguous if they share a common border of at least a quarter of a mile – a form of rook contiguity. Researchers have not been entirely consistent in the form of contiguity they employ.

³The Census Bureau defines the Los Angeles-Long Beach-Santa Ana Metropolitan Statistical Area to include Los Angeles and Orange Counties, and the Los Angeles-Long Beach-Riverside Combined Statistic Area or the Greater Los Angeles Area to include Los Angeles, Orange, Riverside, San Bernardino, and Ventura Counties. When we use the term “Los Angeles metropolitan area”, using small letters for metropolitan area, we are referring to the Greater Los Angeles Area.

three peripheral counties would assert that, to the contrary, there is a well-recognized set of employment subcenters in their county. That GS(20,20) does not identify peripheral subcenters is, of course, due to its defining a subcenter based on *absolute* employment and

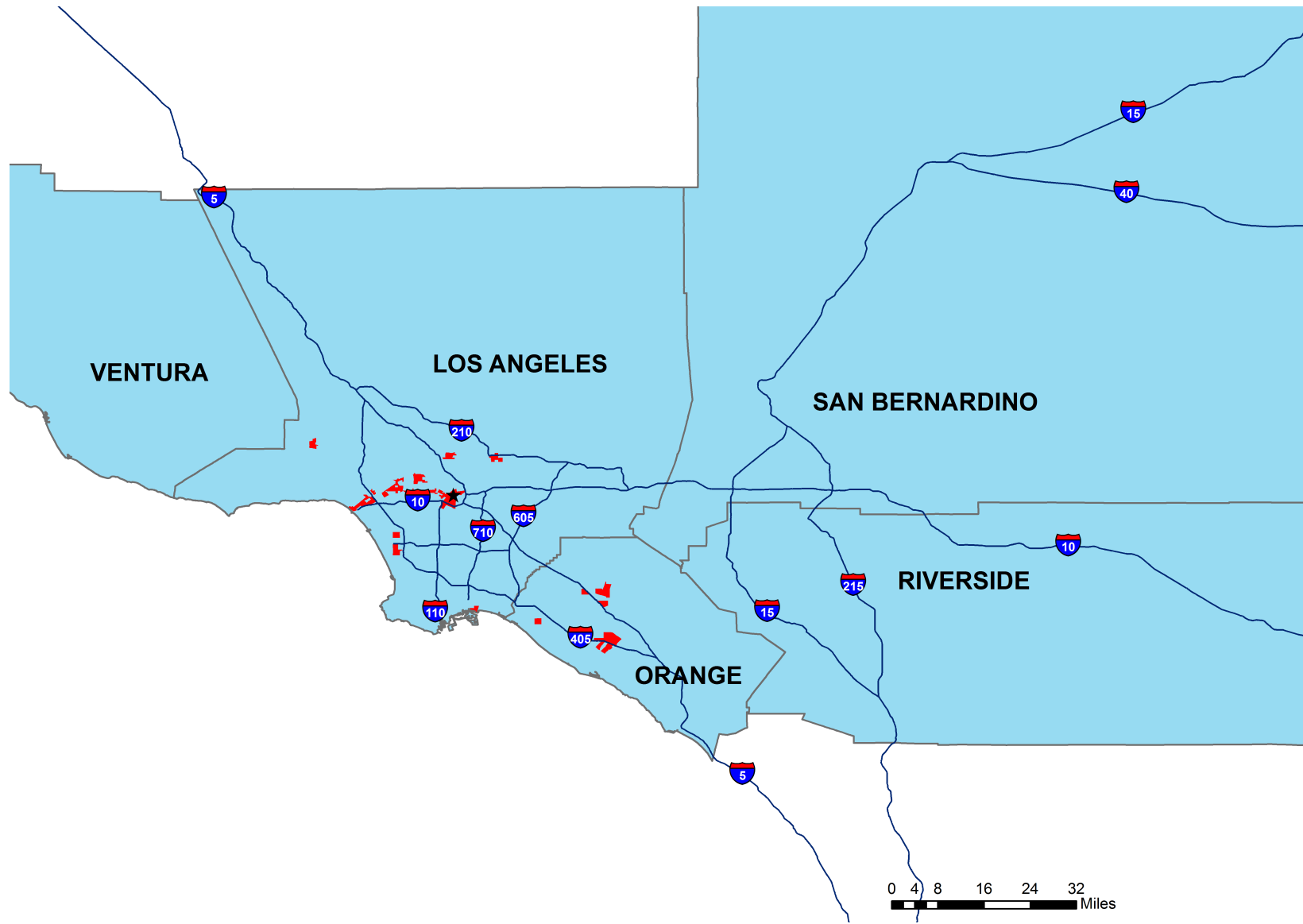


Figure 1: Employment Subcenters in the Los Angeles Metropolitan Area Identified Using GS(20,20) for 2003

Notes: Employment subcenters are shown in red.

★ indicates the metropolitan center, downtown Los Angeles.

absolute employment density.

The aim of this paper is to present an alternative method of identifying subcenters that identifies peripheral, as well as central, subcenters. An obvious alternative approach is to identify subcenters on the basis of employment density relative to that of the surrounding area, and total employment relative to that in the surrounding area. The major problem with this alternative is that all desert communities are then identified as subcenters. Clearly what is needed is some hybrid method that combines absolute and relative employment density and total employment. There are many such methods. This paper explores one that is particularly easy to understand and apply, that is a simple extension of the GS method, and that, at least for the Los Angeles metropolitan area, gives results that seem very reasonable. The general method entails having the employment density and total employment cutoffs fall off in some systematic way with distance from the metropolitan center. The particular method we investigate has the employment density and total employment cutoffs fall off exponentially with distance from the metropolitan center.

We do not think that there is a right method to identify employment (or other types of) subcenters. Different methods are appropriate to different purposes. Later in the paper we shall discuss some possible uses of the method we propose.

Section 1 introduces the alternative method and illustrates its application to a hypothetical city, and then to the Los Angeles metropolitan area in 2003. Section 2 introduces and discusses a refinement of the method, in which the decay rate of cutoff is specified relative to the metropolitan area employment density gradient. Section 3 discusses possible uses of the methods, and Section 4 concludes.

1 The Method of Exponentially Declining Cutoffs

As in the Giuliano-Small method, *an employment subcenter is a set of contiguous zones, each of which has an employment density that exceeds the cutoff employment density for that zone,*

and such that its total employment exceeds the cutoff total employment for its constituent zones. The method of exponentially declining cutoffs differs from the Giuliano-Small method only in that *the cutoffs fall off exponentially with distance from the metropolitan center*.

We employ the following notation:

| | |
|----------|---|
| D | employment density |
| E | total employment |
| α | cutoff gradient |
| x | (Euclidean) distance from the metropolitan center |
| z | zonal index |
| C_Z | set of candidate zones |
| C_S | set of candidate subcenters |

The method of exponentially declining cutoffs has five steps for determining subcenters.

1. Determine the cutoff level of employment density for each zone.

$$\underline{D}_z = \underline{D} e^{-\alpha x_z}$$

where \underline{D}_z is the cutoff employment density in zone z , \underline{D} is the cutoff employment density at the metropolitan center, and x_z is the distance between the zone centroid and the metropolitan center. In words, the cutoff employment density in zone z equals the cutoff employment density at the metropolitan center, adjusted downward as a function of distance from the metropolitan center according to $e^{-\alpha x_z}$.

2. Determine the set of zones that are candidates for inclusion in an employment subcenter on the basis of their employment density, or, in short, the set of candidate zones, which we denote by C_Z .

Where D_z is the employment density of zone z ,

$$z \in C_Z \text{ iff } D_z > \underline{D}_z .$$

In words, a zone is a candidate zone if and only if its employment density exceeds its cutoff employment density based on distance from the metropolitan center.

3. Group zones into candidate subcenters.

A candidate subcenter is a set of candidate zones that form a contiguous set and are contiguous to no other candidate zones. By definition, candidate subcenters are mutually exclusive (i.e., a candidate zone cannot be in more than one candidate subcenter). Let s index the candidate subcenters and C_S denote the set of candidate subcenters. By definition $s \in C_S$.

4. Determine the cutoff level of total employment for each candidate subcenter.

$$\underline{E}_s = \underline{E} e^{-\alpha x_s},$$

where \underline{E}_s is cutoff total employment in candidate subcenter s , \underline{E} is cutoff total employment at the metropolitan center, and x_s is the employment-weighted distance between the candidate subcenter and the metropolitan center. In particular, where n is the number of zones in candidate subcenter s , x_z ($z = 1, \dots, n$) is the distance between zone n and the metropolitan center, and E_z is the total employment of zone z , x_s is defined as $\frac{\sum E_z x_z}{\sum E_z}$. In words, the cutoff level of total employment for a candidate subcenter equals the cutoff level of total employment for a subcenter at the metropolitan center, adjusted downward as a function of distance to the metropolitan center according to $e^{-\alpha x_s}$.

5. Determine the set of subcenters.

Where S is the set of (proper) subcenters, and E_s is the total employment of candidate subcenter s ,

$$s \in S \text{ iff } E_s > \underline{E}_s$$

In words, a candidate subcenter is a (proper) subcenter if and only if its total employment exceeds its total employment cutoff based on distance from the metropolitan center.

We denote this method of exponentially declining cutoffs by $\text{EDC}(D, e, \alpha)$.

Figure 2 illustrates application of this method in an example. The hypothetical metropolitan area contains 23 zones. The table lists each zone's employment, employment density, cutoff employment density based on the distance of the zone centroid from the metropolitan center, and distance of the zone centroid from the metropolitan center (x_z). Zone z 's cutoff employment density is $\underline{D}e^{-\alpha x_z}$. Since in the example $\underline{D} = 15$ and $\alpha = \frac{\ln 2}{40}$, the cutoff employment density is $15e^{-\frac{\ln 2}{40}x_z}$; at a distance of 20.00 from the metropolitan center, the cutoff employment density is 10.61. In the example, there are ten zones whose employment density exceeds the distance-dependent employment density cutoffs, zones 5, 6, 12 (the metropolitan center), 13, 14, 16, 17, 18, 19, and 23, and which are therefore candidate zones. These zones form two mutually exclusive contiguous sets, each of which is a candidate subcenter. The first candidate subcenter (shown as the single hatched area) comprises zones 5 and 6, and the second candidate subcenter (shown as the cross hatched area) zones 12, 13, 14, 16, 17, 18, 19, and 23. A candidate subcenter is a (proper) subcenter if its total employment exceeds its cutoff total employment, calculated as $\underline{E}e^{-\alpha x_s}$, where $\underline{E} = 15000$ is the total employment cutoff at the metropolitan center, and x_s is the employment-weighted distance between the candidate subcenter and the metropolitan center. In the example, the first candidate subcenter is not a proper subcenter, while the second candidate subcenter is a

proper subcenter.

Figure 3 illustrates the set of employment subcenters determined by applying $EDC(20,20, \frac{\ln 2}{40})$ to traffic analysis zones in the Greater Los Angeles Metropolitan Area in 2003. $\alpha = \frac{\ln 2}{40}$ was our initial somewhat educated guess, implying that the density cutoff halves every forty miles. The most noteworthy feature of the Figure is that subcenters appear in the wasteland. 12 subcenters emerge in Riverside County, 7 in San Bernardino County and 1 in Ventura County. With one qualification⁴, these subcenters are what the residents would identify as major employment centers in their respective counties. There are 51 subcenters in the entire metro area, with 19 in Los Angeles County, 11 in Orange County, and 1 on the boundary of Los Angeles county and San Bernardino County.

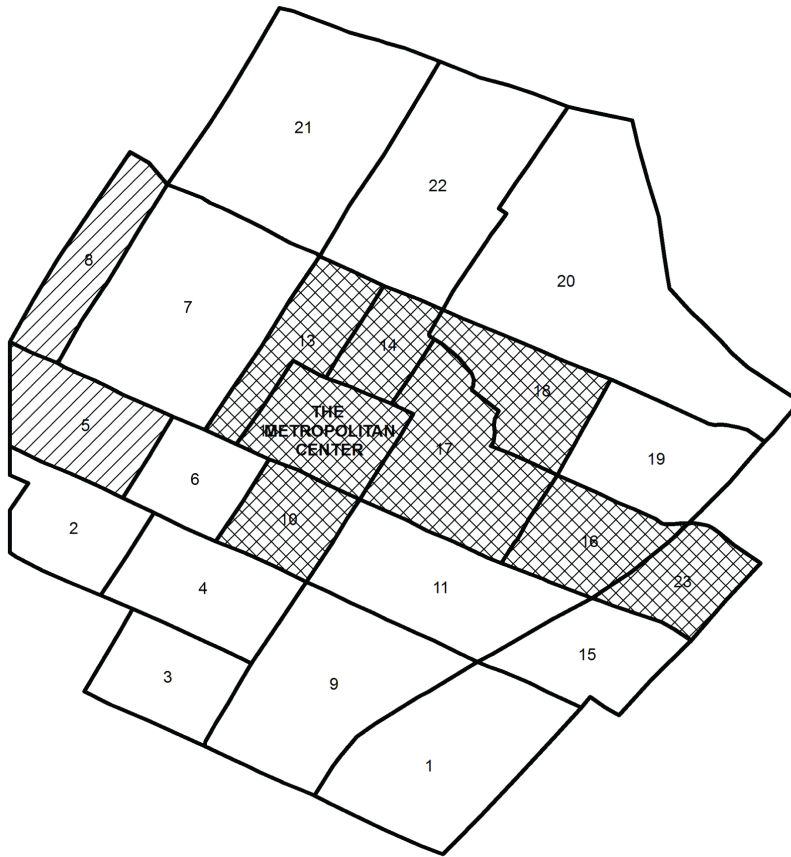
The appendix displays separate maps for each of the constituent counties, and major cities around the subcenters are identified on the map.

At this point, we could show how the set of subcenters changes as the cutoff gradient changes. But we will do that in the next section.

2 A Refinement of the Procedure

That our choice of $\alpha = \frac{\ln 2}{40}$ gives a reasonable balance of subcenters across the six counties is the result of an educated guess. In this section we propose that the employment density gradient for the entire metropolitan area, which we denote by γ , be pivotal in the choice of α . The employment density gradient gives the average exponential rate at which employment density falls off with distance from the metropolitan center. If α is set equal to γ , then a zone is identified as a candidate zone by its employment density relative to the mean (fitted) employment density at that distance from the metropolitan center. In this sense, candidate

⁴The one qualification concerns the Palm Springs area. First, it is reasonable to argue that Palm Springs is part of the Greater Los Angeles Metropolitan Area only by definition and should properly be considered a separate metropolitan area. Second, most Riverside County residents would view the Palm Springs area as a single employment center. The reason it comprises several subcenters derives from its land use. Being hemmed in by mountains, it is long and narrow, and along the corridor predominantly commercial zones alternate with predominantly residential zones.



| ID | employment | empdens | cutoff | dist_center |
|----|------------|---------|--------|-------------|
| 1 | 664 | 7 | 12.87 | 8.83 |
| 2 | 410 | 9.27 | 13.52 | 5.99 |
| 3 | 221 | 5 | 13.23 | 7.25 |
| 4 | 400 | 7.04 | 13.79 | 4.86 |
| 5 | 870 | 17.06 | 13.69 | 5.29 |
| 6 | 278 | 7.78 | 14.22 | 3.07 |
| 7 | 1547 | 12.59 | 13.91 | 4.35 |
| 8 | 866 | 21.38 | 13.35 | 6.72 |
| 9 | 1210 | 12.22 | 13.36 | 6.67 |
| 10 | 611 | 17.99 | 14.37 | 2.46 |
| 11 | 500 | 6.49 | 13.84 | 4.65 |
| 12 | 9000 | 193.34 | 15 | 0 |
| 13 | 6400 | 161.58 | 14.4 | 2.37 |
| 14 | 2100 | 79.58 | 14.35 | 2.55 |
| 15 | 300 | 6.41 | 13.02 | 8.16 |
| 16 | 2600 | 61.61 | 13.46 | 6.26 |
| 17 | 2100 | 27.43 | 14.26 | 2.92 |
| 18 | 2125 | 44.34 | 13.9 | 4.38 |
| 19 | 177 | 2.97 | 13.26 | 7.11 |
| 20 | 920 | 5.19 | 13.26 | 7.13 |
| 21 | 930 | 7.32 | 13.09 | 7.84 |
| 22 | 540 | 4.94 | 13.32 | 6.83 |
| 23 | 660 | 19.31 | 12.92 | 8.61 |

Figure 2: An Example Application of the Method of Exponentially Declining Cutoffs

Notes: $\underline{D} = 15$, $\underline{E} = 15000$ (so that $e = 15$), and $\alpha = \frac{\ln 2}{40}$.

The single hatched area is a candidate subcenter but not a (proper) subcenter. The cross-hatched area is a (proper) subcenter.

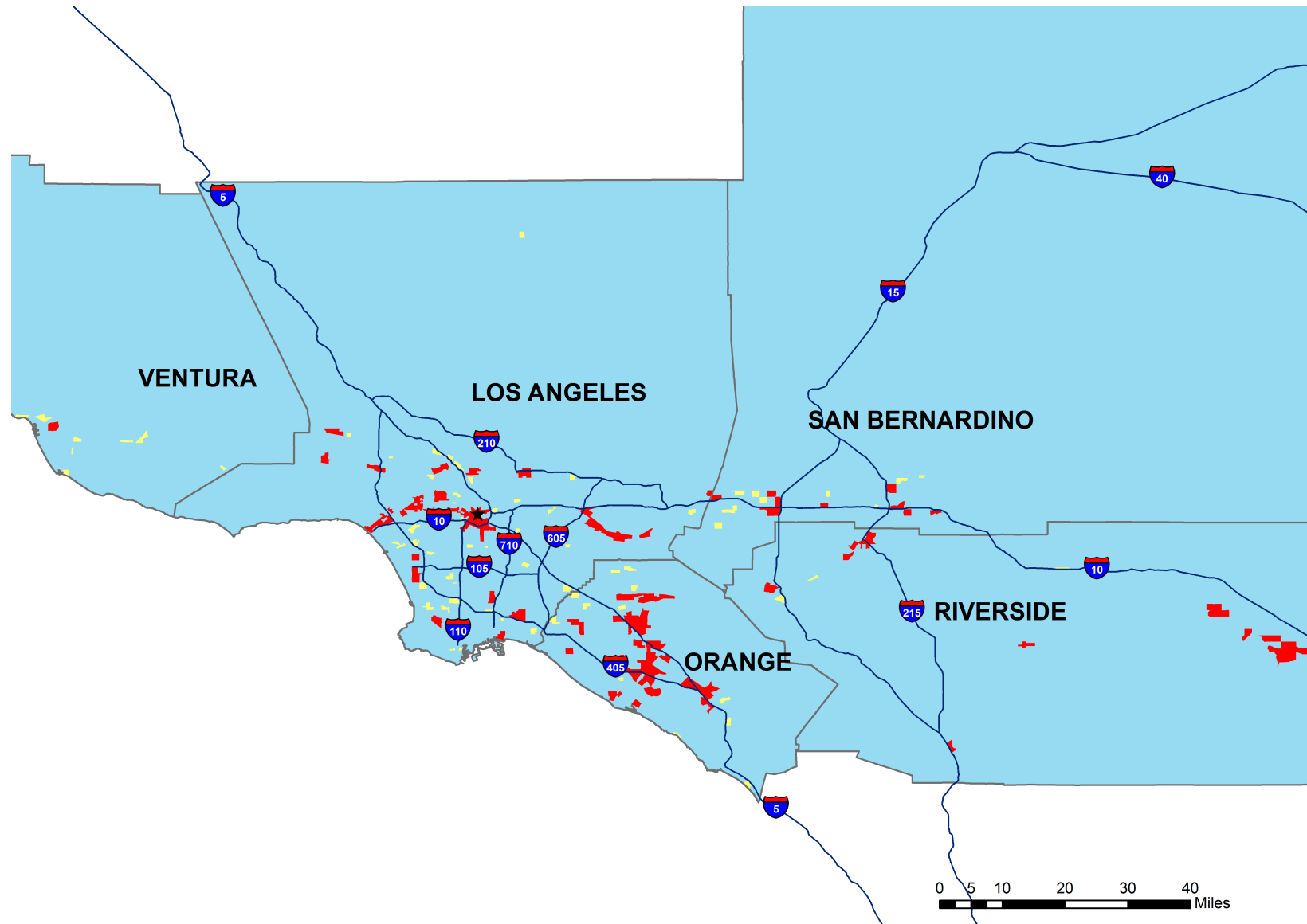


Figure 3: Employment Subcenters in the LA Metro Area Identified Using $EDC(20,20, \frac{\ln 2}{40})$

Notes: A yellow area indicates a candidate subcenter that is not a proper subcenter. A red area indicates a proper subcenter.

zones are identified by their relative employment densities⁵ (more specifically, relative to the fitted employment density at that distance from the metropolitan center). More generally, one can set $\alpha = \theta\gamma$, where θ measures the weight attached to relative employment density compared to absolute employment density. When $\theta = 0$, all the weight is attached to absolute employment density, and our method reduces to the corresponding GS method. When $\theta = 1$, all the weight is attached to employment density relative to the fitted employment density at that distance from the metropolitan center⁶. We define $DGC(D, e, \theta; \gamma)$ to be the *method of density-gradient-related cutoffs*. The method is characterized by three parameters, D , e , and θ , in addition to a metropolitan area’s employment density gradient.

How should the employment density gradient be estimated? The *simple employment density gradient* is the estimated value of γ by OLS in the regression equation:

$$\ln e_z = c - \gamma x_z + u_z$$

where z indexes the zone and u_z is the error term (Mills and Tan, 1980). More sophisticated estimates of the employment density function can be obtained by adding other accessibility co-variates, such as distance from the nearest freeway and, in the case of the Los Angeles metropolitan area, distance from the ocean (but not distance from nearby subcenters, since they are endogenous to the procedure), by taking account of spatial correlation in the error term, and by using more flexible functional specifications and non-parametric specifications. We favor the use of the simple employment density gradient since its estimation follows a standard procedure. In contrast, if non-standards method were used, different studies would employ different sophisticated methods, which would make comparability across studies more difficult.

Using traffic analysis zones, the estimated value of γ for the five-county Los Angeles

⁵Note that this is a different measure of relative employment density than that discussed earlier, which looks at the employment density of a zone relative to that of neighboring zones.

⁶Values of θ greater than one are possible, but we cannot think of situations where one would want to identify subcenters by having the employment density cutoff fall off at a faster rate with distance from the metropolitan center than the metropolitan area’s employment density gradient.

Metropolitan Area in 2003 is 0.03172. Figure 4-1 shows the subcenters identified according to DGC(20,20,1) – the Giuliano-Small method but with cutoffs declining exponentially at the rate of the simple employment density gradient. To our eyes, applying the DGC method with $\theta = 1$ to the LA Metro Area identifies too many subcenters, especially at the metropolitan periphery. The reason seems to be that employment is more spatially concentrated at the metropolitan periphery, lying close to the freeways. Figure 4-2 shows the subcenters identified according to DGC(20,20,0.5) – the Giuliano-Small method but with the cutoffs declining exponentially at a rate equal to one-half of the simple employment density gradient. The employment subcenters applying the DGC method with $\theta = 0.5$ are almost the same as those identified in Figure 3. The reason is that the rate of exponential decay in Figure 3, $\frac{\ln 2}{40} = 0.01732$, is similar to 0.5 times the density gradient of 0.03172, 0.01586. Thus, applying the DGC method to the LA Metro Area with $\theta = 0.5$ gives reasonable results. Whether this generalizes to the other metropolitan areas remains to be seen.

3 Discussion

There is much to be said for the GS method of employment subcenter identification. It is intuitive, easy to implement, and robust. There is also much to be said for continuing to use the standard method, whatever it is, since it facilitates comparison of results across time and metropolitan areas. Since it is not difficult to come up with methods that are superior to the GS method in *some* respects (see, for example, Giuliano et al., 2007; McMillen, 2001; and Redfearn, 2007), there is also the danger of method proliferation if alternative methods start to be used, which would compromise the comparability of results across studies. Thus, an alternative method needs a strong justification to be considered.

In this paper we have put forward an alternative method of subcenter identification – the method of declining exponential cutoffs. The method is a generalization of the GS method, and has the GS method as a limiting case. The justification for the generalized method is,

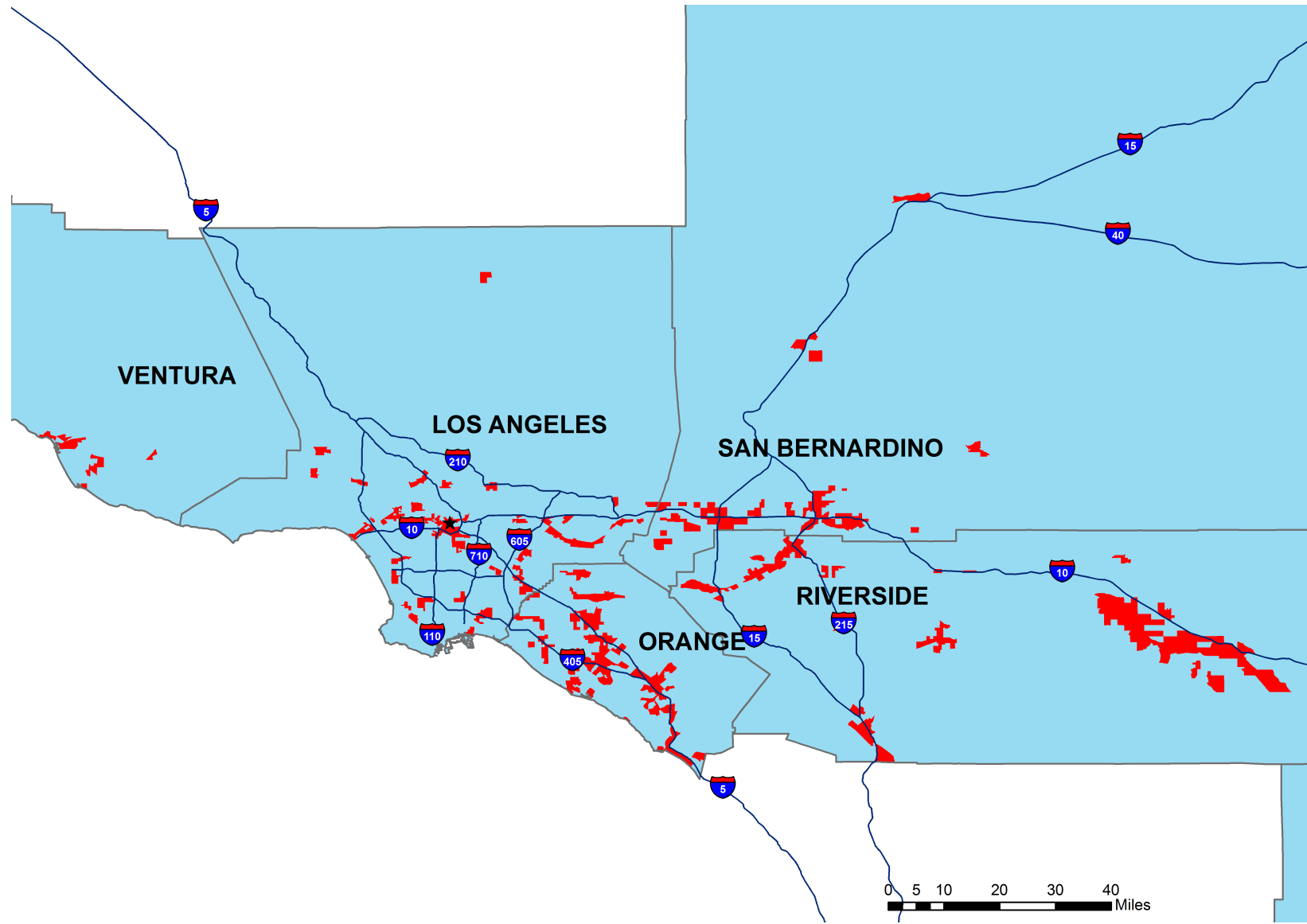


Figure 4-1: Employment Subcenters in the Los Angeles Metropolitan Area Identified Using DGC(20,20,1) for 2003

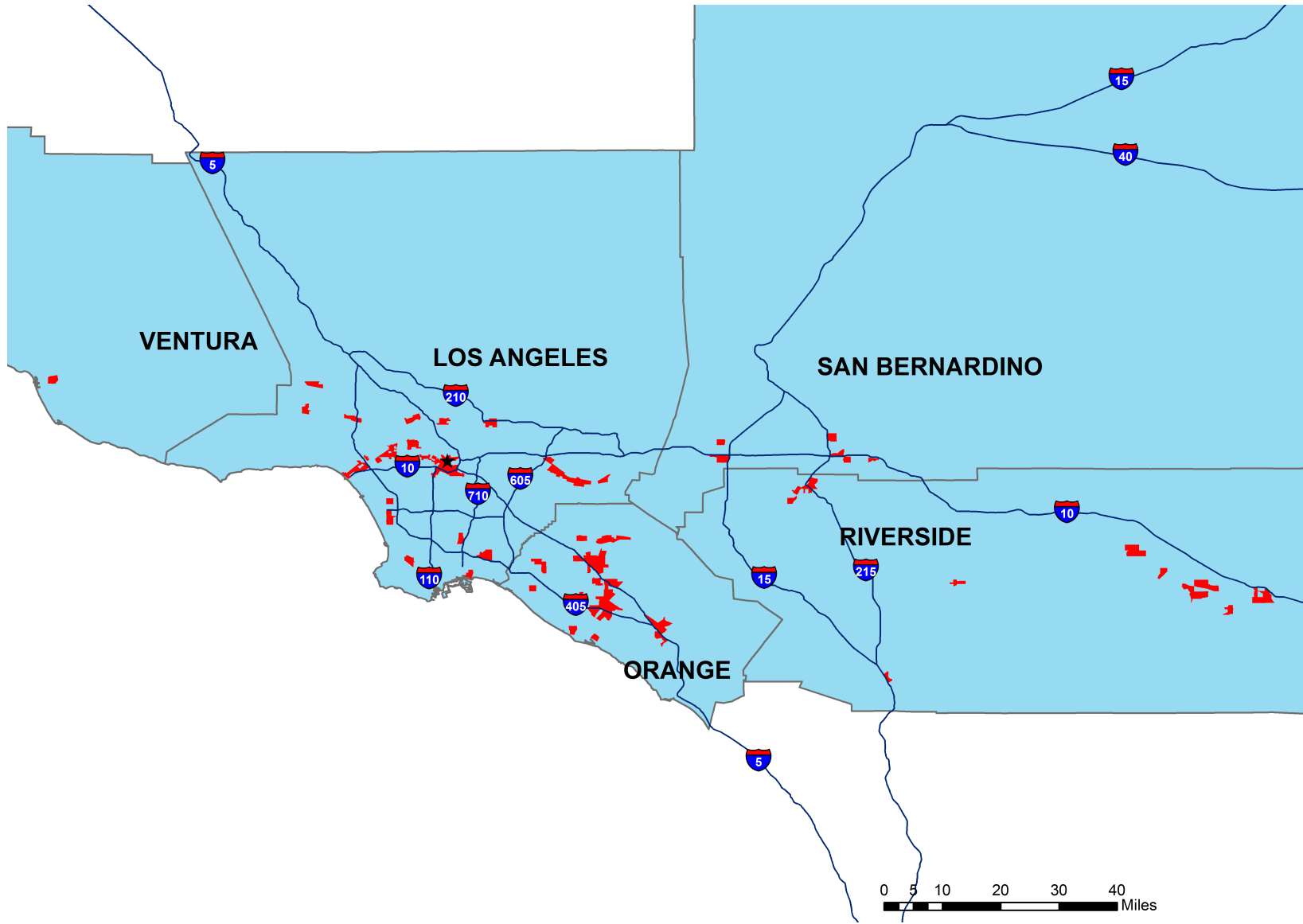


Figure 4-2: Employment Subcenters in the Los Angeles Metropolitan Area Identified Using DGC(20,20,0.5) for 2003

we think, a strong one. Being based on absolute employment density and absolute total employment, the GS method may fail to identify many important subcenters in the suburbs and exurbs. The Los Angeles metropolitan area is an extreme case, since application of the GS(20,20) method for 2003 identifies no subcenters at all in the three peripheral counties, which in 2000 had a combined population of four million. Maps strongly influence the way we perceive space. According to the GS employment subcenter map, the periphery of the Los Angeles metropolitan areas is an undifferentiated wasteland. But that is not at all the perception of those counties' residents.

There are many planning contexts in which identification of employment subcenters in the periphery of metropolitan areas is important and valuable⁷. The most compelling relate to elements of the new urbanism/smart growth planning movement, with its emphasis on mixed-use centers and transit corridors, among other planning instruments, as ways to combat urban sprawl and to encourage greener lifestyles. At the metropolitan periphery, it is good sense to make existing and emerging subcenters the mixed-use centers, and also to have the transit corridors join existing and emerging subcenters. But the GS method provides little or no guidance on how to identify these subcenters at the urban periphery.

One can reasonably argue that there are other, simpler ways of extending the GS method to identify peripheral subcenters than the method this paper proposes. Indeed, we have experimented with alternative methods. The most obvious is to lower the cutoffs. The method has two disadvantages. First, if one applies the GS method with lower cutoffs to the entire metropolitan area, the central area in subcenters increases, through subcenters increasing in size and melding and through the emergence of new subcenters. Alternatively, if one applies the GS method with higher cutoffs to central areas and with lower cutoffs to peripheral areas, problems arise at the boundary between the central and peripheral areas. Second, whichever of the two sub-methods is employed, the same "wasteland" problem arises

⁷Indeed, the original motivation for this line of work was a complaint from the planning staff at the Southern California Association of Governments (SCAG, which is the metropolitan planning agency for the Greater Los Angeles Area and Imperial County) about the failure of the GS method to identify subcenters at the metropolitan periphery.

but on a smaller scale. Exurbia becomes a wasteland from the perspective of suburbia. Another alternative method is to identify peripheral subcenters on the basis on population forecasts, which entails identifying emerging subcenters. This method in fact worked quite well using earlier (pre-meltdown) SCAG population forecasts. But, at least in the United States, population forecasting at the intra-metropolitan level is politicized since some grant allocations are based on them. Identifying subcenters on the basis of population forecasts can only be as sound as the underlying forecasts.

Identifying employment subcenters in the metropolitan periphery is important in other contexts too. An obvious application is to transportation planning, whether or not new urbanist thinking heavily influences the planning goals. We derived the method for quite a different purpose, for zonation. We were asked to assist in dividing the SCAG (Southern California Association of Governments) Region (the five counties of Greater Los Angeles, plus Imperial County) into 100 zones for the application of a dynamic computable general equilibrium model of land use, transportation, and environmental quality, RELU-TRAN (Anas and Liu, 2007), to the Los Angeles Metropolitan Area. The original proposal was to build zones around randomly located seeds. We proposed instead that the zones be built around employment subcenters. This would not have worked well if we had employed the GS method since San Bernardino and Riverside Counties would have been a single zone, so we developed our method of exponentially declining cutoffs⁸. We are considering applying our method to identifying alternative types of subcenters, such as retail subcenters, floor area subcenters, and trip subcenters. Comparison of the spatial pattern of the different types of subcenters, according to methods that identify subcenters at the metropolitan periphery should prove to be a useful planning tool. For example, transit-oriented development is more likely to be successful if there is considerable spatial overlap of subcenter types.

⁸It turned out that our method, by itself, had a weakness in this application. As D and e were lowered, holding α fixed, the numbers of subcenters initially grew, but reached a maximum, and then started to decline. As expected, new subcenters emerged, but also existing subcenters melded, particularly along freeways. The 100 zones were obtained by having both subcenter seeds and random seeds. (See Church and Li, 2010 for a detailed description of the actual method employed).

Thus, we think that the case for employing exponentially declining cutoffs in identifying peripheral subcenters is compelling in many *intra-metropolitan* planning contexts. We do not however advocate use of the method presented in this paper for *inter-metropolitan* comparisons. GS works well in comparing the number of subcenters above certain absolute sizes across metropolitan areas. The EDC method however would not work well in cross-metropolitan comparisons since the rate at which employment density falls off with distance from the metropolitan center varies substantially across metropolitan areas, even in the United States (see Glaeser and Kahn, 2001). In the Los Angeles metropolitan area, we found that halving the cutoffs every forty miles from the metropolitan center succeeded in achieving a nice balance between central and peripheral subcenters, but, if the same rule were followed in other metropolitan areas, in some of them most of the subcenters would be close to the downtown core while in others most of the subcenters would be at the metropolitan periphery.

A related issue concerns which should be employed, the EDC method, in which the exponential rate at which the cutoffs decline is specified exogenously, or the DGC method, in which the exponential rate at which the cutoffs decline is related to the metropolitan area's employment density gradient. Both methods entail the same steps to apply. The latter method has the advantage that it provides a reference point in the choice of the exponential rate to apply. Even so, there is no right method for choosing subcenters. Which method gives results that seem more satisfactory depends on context, and for a particular method which exponential rate gives results that accord better with intuition is a matter of experimentation. Furthermore, there is insight to be gained from experimenting with different methods and different exponential rates.

4 Concluding Comments

Metropolitan spatial structure is complex. Viewing this complexity through the lens of identifying subcenters is one way of making this complexity more comprehensible. The standard method of identifying subcenters, the Giuliano-Small (GS) method, is intuitive, robust, and simple to apply, and is a sound method for comparing one aspect of the spatial structure of different metropolitan areas. Employing a standardized method has the considerable advantage that it permits comparability of results over time and across metropolitan areas. Deriving new and more sophisticated methods of subcenter identification is not difficult. But new methods should be embraced with caution since proliferation of methods undermines the comparability of results across studies.

Even with this conservative caveat in mind, we believe that this paper's extension of the GS method merits serious consideration for adoption in some intra-metropolitan applications. When applied to the Los Angeles metropolitan area in 2003, the GS(20,20) method identifies no subcenters in the peripheral counties of Riverside, San Bernardino, and Ventura, despite their having a combined population at the time of 4 million and despite their having well-defined spatial structures, albeit on a finer scale. The reason is simply that the GS method identifies subcenters on the basis of absolute employment density and absolute total employment. But in some planning contexts it is desirable to identify subcenters giving some weight at least to employment densities and total employment relative to averages in the locality. The method we presented entails the employment density cutoff and the total employment cutoff of the GS method falling off exponentially with distance from the metropolitan center. We illustrated that application of our method yields sensible and intuitive results, identifying correctly what local residents would view as the peripheral subcenters. We also proposed a refinement that allows different weights to be attached to relative and absolute in the choice of cutoffs. With a weight of zero, the method reduces to the GS method. With a weight of one, the exponential rate at which the cutoffs fall off with distance from the metropolitan center equals the metropolitan area's employment density gradient.

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Appendix

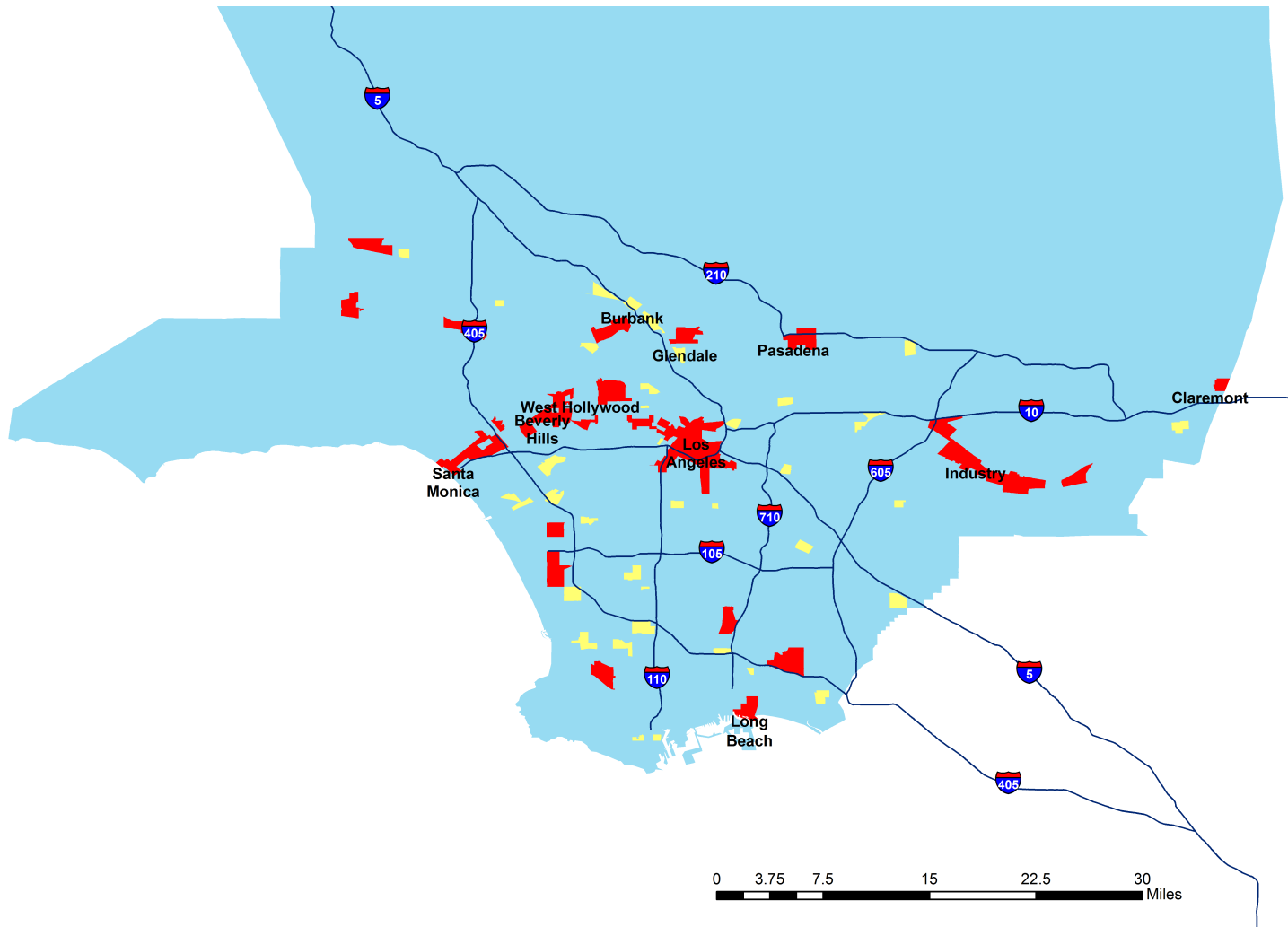


Figure A1: Employment Subcenters in Los Angeles County Identified Using EDC(20,20, $\frac{\ln 2}{40}$) for 2003

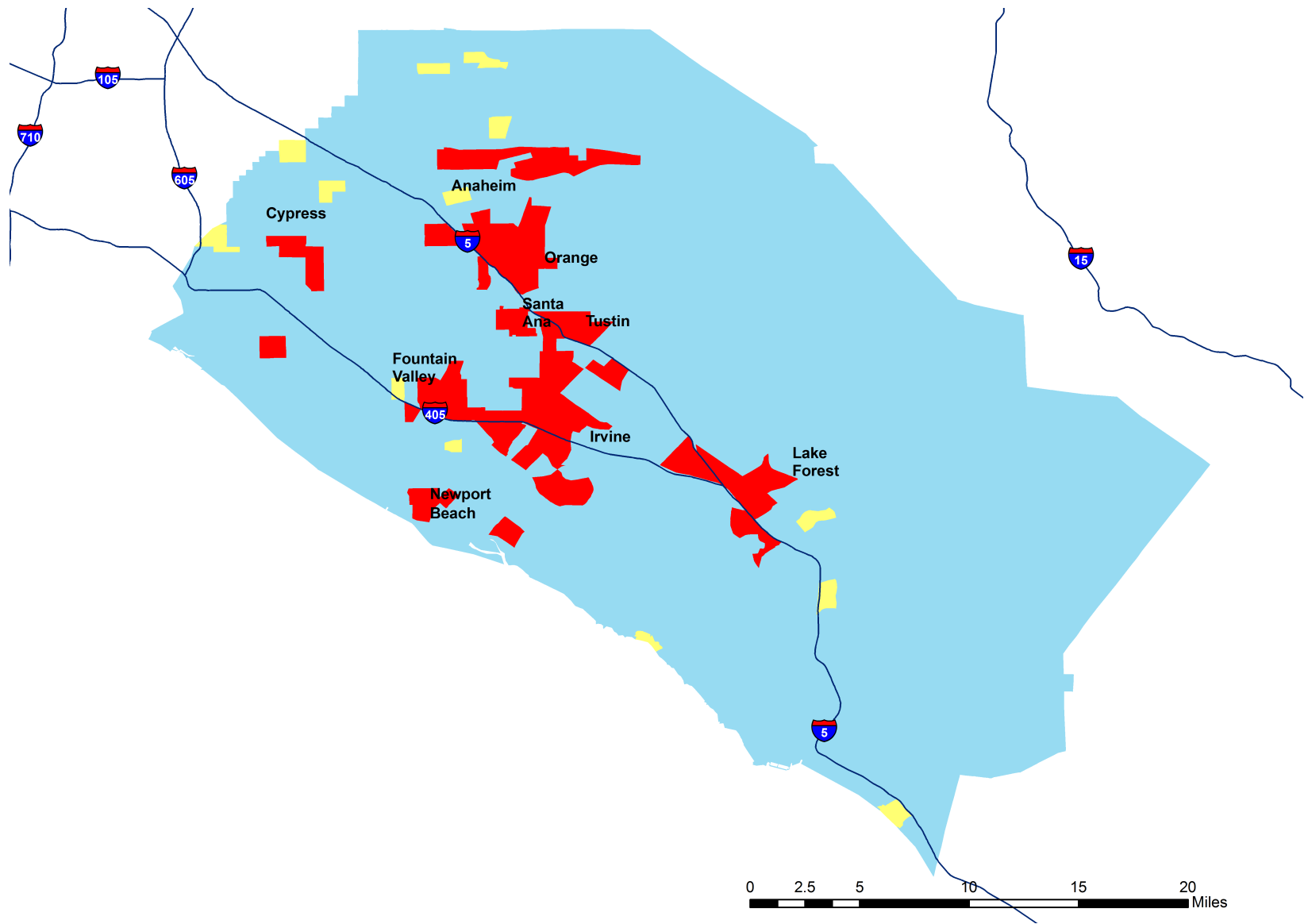


Figure A2: Employment Subcenters in Orange County Identified Using $EDC(20,20, \frac{ln2}{40})$ for 2003

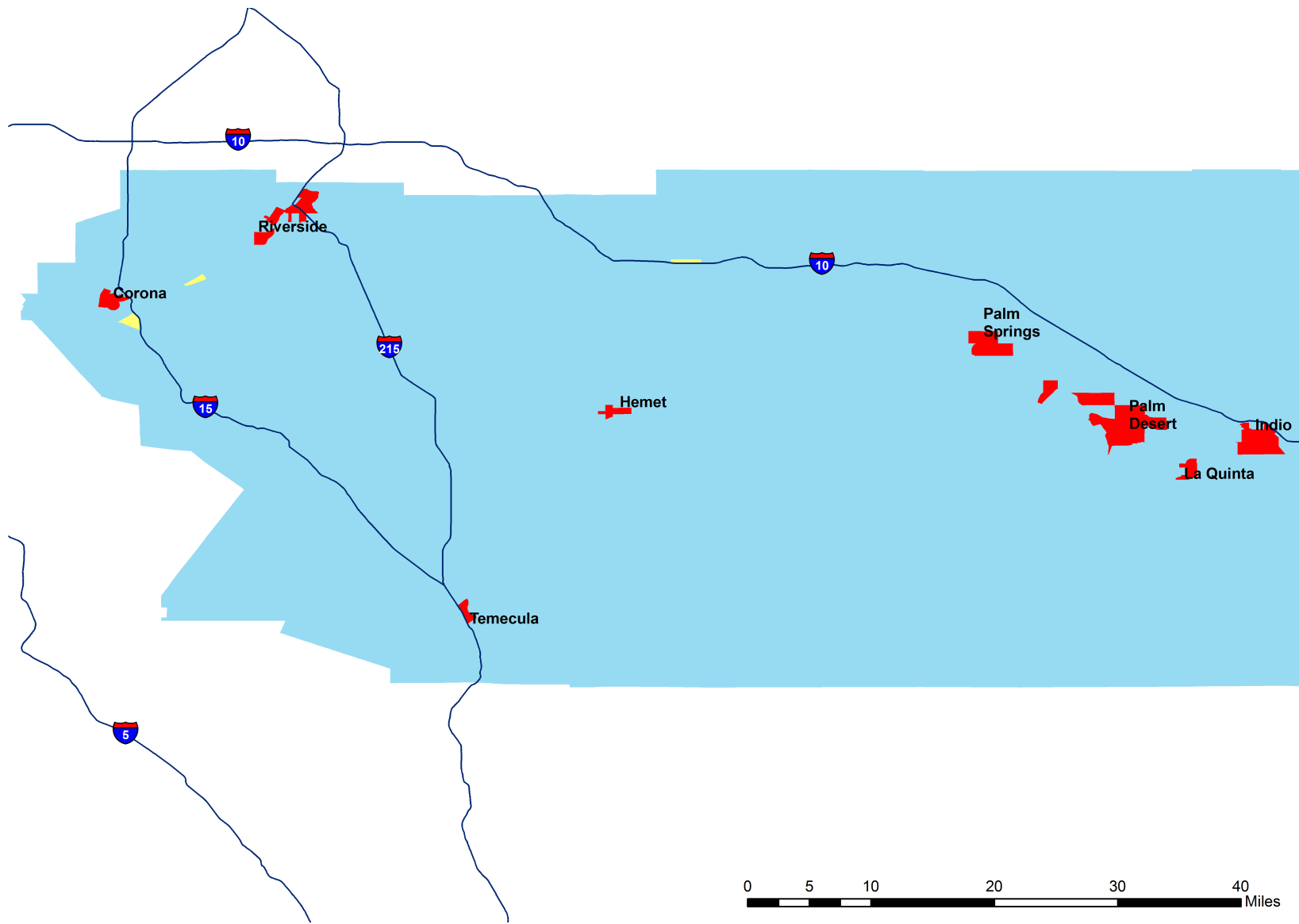


Figure A3: Employment Subcenters in West Riverside County Identified Using $EDC(20,20, \frac{\ln 2}{40})$ for 2003

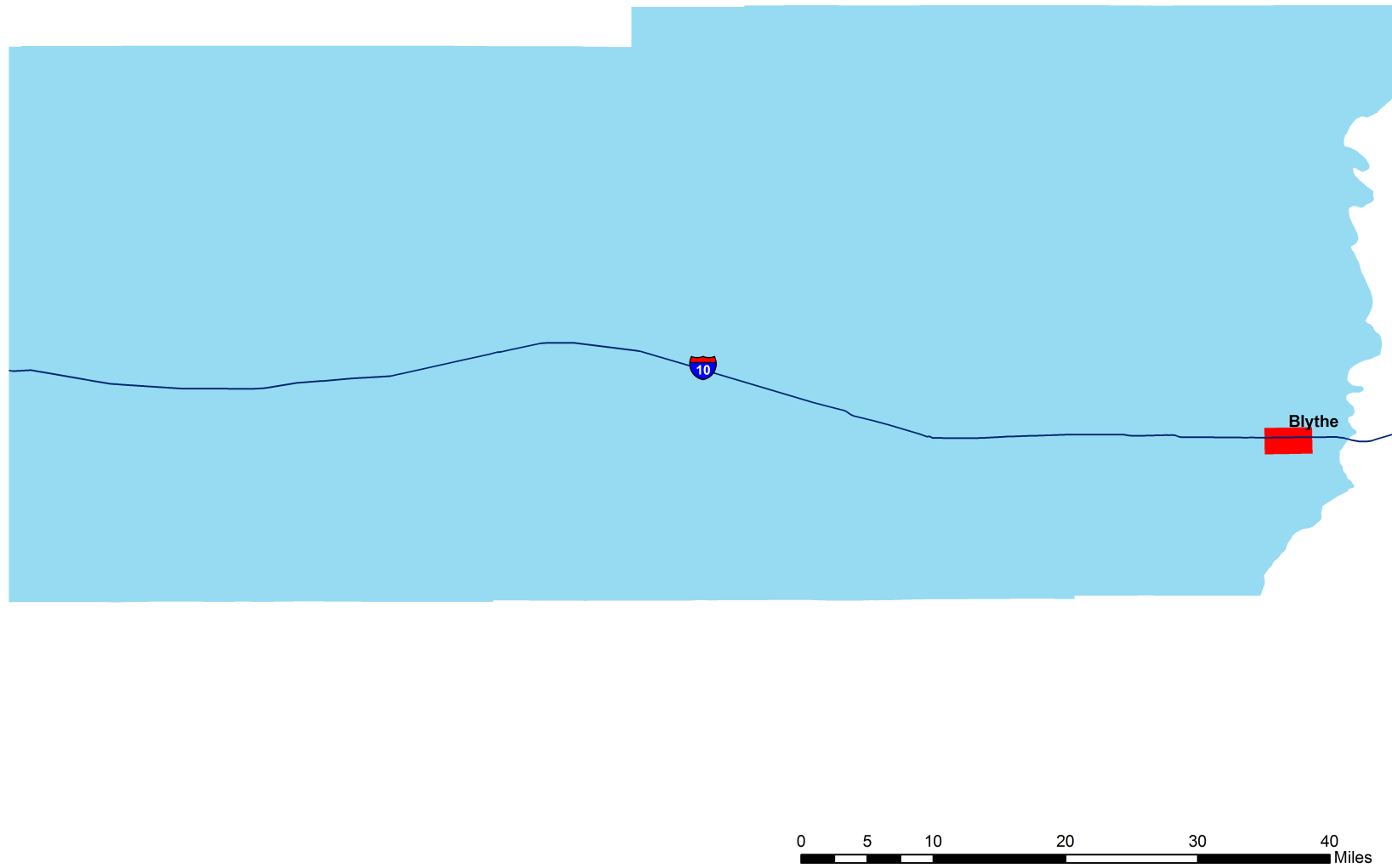


Figure A4: Employment Subcenters in East Riverside County Identified Using $EDC(20,20, \frac{\ln 2}{40})$ for 2003

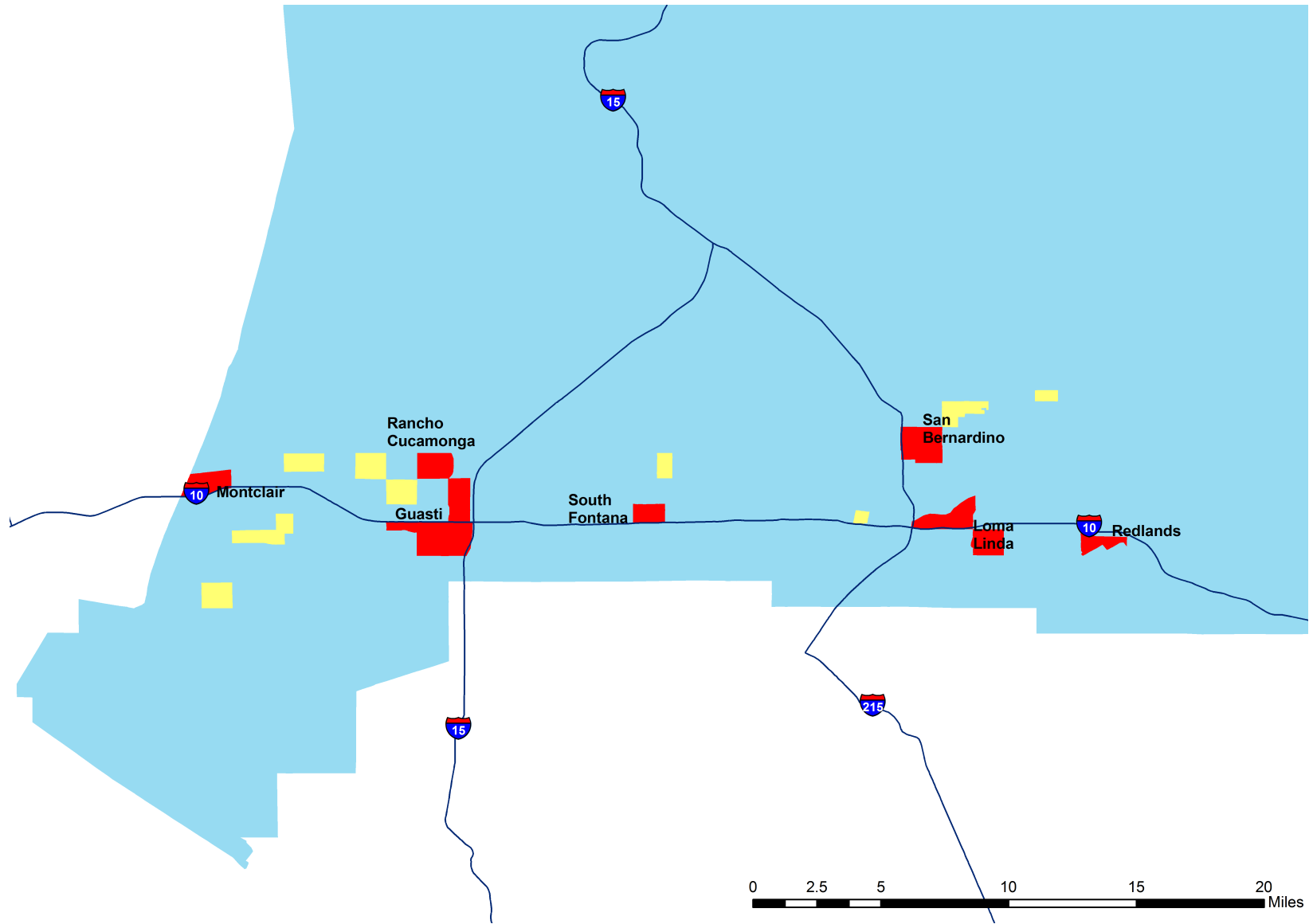


Figure A5: Employment Subcenters in San Bernardino County Identified Using $EDC(20,20, \frac{\ln 2}{40})$ for 2003

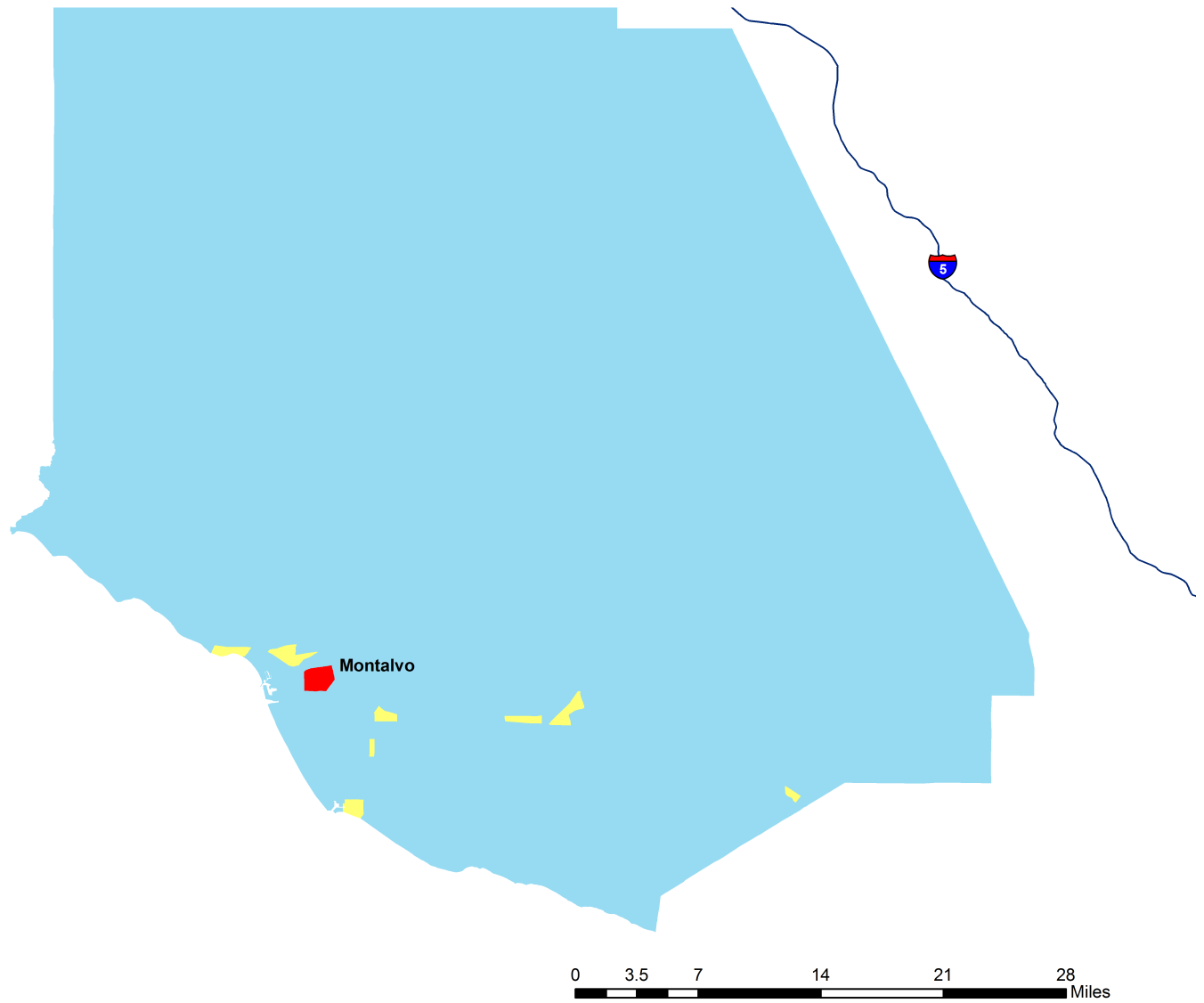


Figure A6: Employment Subcenters in Ventura County Identified Using $EDC(20,20, \frac{ln2}{40})$ for 2003